TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 308

LAW OF SIMILITUDE
FOR THE SURFACE RESISTANCE OF LACQUERED PLANES
MOVING IN A STRAIGHT LINE THROUGH WATER.

By Friedrich Gebers.

Translated from "Schiffbau,"
Vol. 22, 1921, Nos. 29, 30, 31, 32, 33, 35 and 37.

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E R R A T A

On Title Page substitute
"Translated from 'Schiffbau,'
Vol. 22, 1921, Nos. 29, 30, 31, 32, 33, 35 and 37,"

for

"Paper read by Dr. G. Kempf, etc."

so as to make Title Page read:

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CONTENTS

Preface

The present article contains the results of scientific research during the last years of the war, which research was planned on a considerably larger scale, as yet only partially carried out, and whose continuation and conclusion cannot yet be foreseen.

This paper was originally prepared as a lecture before a gathering of specialists, but was then requested for official publication. It was accordingly revised and enlarged and, after completion, on account of being too voluminous, was released for publication in some other way. This explains the nature of the composition and the addition of the appendix. I shall now leave it as it is and let it go forth into the world as proof that, even in times of direst need, sanctuaries were provided for scientific research.

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1. Introduction; Earlier experiments.

During the last few years, many articles have been published on the law of similitude as applied to the phenomena of friction in fluids (see Appendix). We will here call attention only to the articles published by Blasius,* by Gümbel** and by

*"Das Aehnlichkeitsgesetz bei Reibungsvorgängen in Flüssigkeiten," Zeitschrift des Vereins Deutscher Ingenieure, 1913, No. 131.
**"Das Problem des Oberflachenwiderstandes," Jahrbuch der Schiffbautechnischen Gesellschaft, 1913, p. 393.

Stanton and Pannel.* Other references will be found in these articles.

Aside from the foregoing, the best known investigators in this field are Saph and Schoder, Darcy, Nusselt, Reynolds and Lang, though the list could be considerably extended.

While the law of similitude has been abundantly confirmed for tubes, its applicability to flat surfaces has, hitherto, not been verified by a single investigator, although the theory undoubtedly applies.

The reason lies primarily in the much greater difficulty and cost of such experiments; secondly, in the fact that the results of the classical experiments of Froude** had been universally accepted; thirdly, in the fact that only in recent times we have gradually come to recognize the applicability of the law of similitude to fluids of various viscosities; and, lastly, in the fact that the constantly increasing accuracy of the experiments enables the introduction of new factors (e.g., even a slight variation in the temperature of the fluid) into the computations.

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* "Similarity of Motion in Relation to the Surface Friction of Fluids," Philosophical Transactions of the Royal Society of London, Series A, Vol. 214, 1914, page 199.

** W. Froude, "Experiments on Surface Friction Produced by a Plane Moving through Water," a paper read before the British Association at Brighton in 1872.

W. Froude, "Report to the Lords Commissioners of the Admiralty on Experiments for the Determination of the Frictional Resistance of Water on a Surface under Various Conditions, performed at Chelston Cross, under the authority of their Lordship, read at Belfast, 1874.

Perhaps the only experiments which have been published, since Froude's famous reports, are those published by the writer in 1908.* These Dresden-Uebigauer experiments covered only a small range, in comparison with Froude's experiments, but were otherwise very similar. They were only intended to furnish the coefficients for a certain coat of paint on wooden models by means of Froude's formulas for the determination of a ship's resistance from experiments with models. They contained the same errors as Froude's experiments, namely, those due to the neglect of the water temperature and the termination of both ends of the wooden planes by smooth, sharp brass plates. Moreover, the planes were not similar in their dimensions, but the shorter plane was made from the longer plane, by simply cutting off a portion of the latter and replacing the sharp-edged strip of brass. Hence, experiments could not be repeated with the long planes. These experiments were executed, however, with great care and yielded somewhat smaller values than those of Froude, although the planes had a greater thickness (8 mm instead of 4.8 mm). For the evaluation of the results, the smaller resistance of the two brass strips, towed alone through the water, was replaced by the greater resistance of a lacquered sharp steel plate of 2 mm thickness, which was not done by Froude.

Blasius attempted to ascertain the law of similitude from these results with dissimilar planes and, although he found con-

* Gebers, "Ein Beitrag zur experimentelen Ermittlung des Wasserwiderstandes gegen bewegte Körper," Zeitschrift "Schiffbau," Vol. IX, Nos. 12 and 13.

siderable agreement with the theoretical considerations, there still remained some uncertainty, on account of the scattering of the points and especially in the introduction of the temperature of the water.

2. Apparatus.

In 1914, at the suggestion of the writer, the management of the "Schiffbautechnische Versuchsanstalt" in Vienna decided to institute a series of experiments for determining the law of similitude. It was hoped, through the much greater available velocity than in Froude's experiments and through improvements in the apparatus, to make permanent progress in the solution of the whole problem. The new institute did not at first expect much business in the form of paid towing experiments, but considerable time for scientific research. The originally very imperfect apparatus would not have sufficed, however, for many other scientific investigations, even though very alluring.

The founder of the Vienna Institute and the President of the Experimental Department, Dr. Wilhelm Exner, who took an active part in raising the considerable amount of money required, deserves great credit for the realization of the experiments.

Since the Institute was expected to begin operations early in 1916, preparations for the experiments were initiated in the summer of 1915. This was fortunate, for it would have been hardly possible to obtain the necessary materials later.

Previous experiments had indicated that the stiffest, most homogeneous wood was about the only suitable material for the production of long planes for towing in the vertical position in water. California redwood was chosen and a few suitable timbers were fortunately found, which were cut into the desired form. The dimensions of the planes had to be governed by the dimensions of the timbers. It was possible to make a plane 10 m long and 0.5 m wide by simply fitting two boards together. Each plane was provided with a lead keel (the same as in Froude's experiments), which held it vertical at just the right depth. In the Dresden experiments, however, the planes projected a little above the water, instead of being towed entirely submerged, as in Froude's experiments. This was done to eliminate one of the edges, on account of the possibility that the resistance on the edges might differ from that on the sides.

In order to obtain the maximum uniformity, the ends of the plane were provided with wedge-shaped tips made of sheet brass. If the ends of the wooden plane itself were sharpened, the tapered portion would be uneven and would, moreover, be very easily damaged. Preliminary experiments had demonstrated that sheet brass and also lead could be rendered smoother with lacquer, which was accordingly spread over the whole surface.

Planes having the following dimensions were employed.

Table I (See also Fig. 3).

a	b	c	d	e	f	g	h
Plane No.	Length incl. tapered ends m	Height incl. lead keel mm	Thick- ness mm	Depth sub- merged mm	Length of end sections mm	Length of tapered portions mm	Height of end sections mm
I	1.25	70*	2.5	62.5	60	50	112.5
II	2.5	135	5	125	120	100	175
III	5	275	10	250	240	200	350
IV	7.5	410	15	375	360	300	500
V	10	535	20	500	480	400	650

* Had no lead keel.

In addition to the above, there were prepared: one plane 5 m long, 10 mm thick, with a submersion depth of 500 mm; and two planes for lengthening the 10 m plane to 15 m and 20 m respectively. The wider 5 m plane was expected to show whether the specific resistance varies with the height of the surface. For the same purpose, a varying submersion of plane III was also planned. The tapered end sections had to be made wider than the rest of the plane, to allow for attaching the towing device.

The wood was exceptionally straight-grained and free from knots. It proved to be much more suitable than the pine wood employed in the Dresden experiments. The planes neither warped nor dished. The lacquer spread easily and uniformly on the wood, which had been previously planed and soaked with linseed oil. The specific gravity of the unvarnished wood was found, by weighing, to be 0.3925, for a plane 10 mm thick, and 0.3967 for a plane 20 mm thick. The specific gravity of the lead was taken as 11.6 and the height of the lead keel was so calculated that the planes, with the extra weight of the tapered ends, sank a little deeper than was desired for the experiments. They could then be easily adjusted in attaching to the towing device. The basin for the experiments had a width of 10 m, a length of 180 m, and a water depth of fully 5 m. The maximum speed of the towing car, which ran above it, was normally 7.5 m/s and this could be increased to 8.5 m/s by overloading the motors. Though the latter speed was employed without apprehension in the first experiments, it was subsequently discontinued, on account of the great increase in the cost of repairs and the difficulty in getting them made promptly, due to the continuance of the war. As measuring instruments, we had the Institute's resistance dynamometer and a suspension device for the front tapered section of the plane (Fig. 1). With the aid of an auxiliary spring, it measured up to 70 kg. For greater forces, a special device (Fig. 2) was constructed, which, with the aid of the dynamometer, could measure up to 200 kg. Only ball and knife-edge bearings were employed. The rear tapered section of the plane was suspended by means of a small steel wire about 2.5 m long, from the middle girder of the car or from an extension of the same. The dynamometers balanced thereby in their middle position. The calibration was made by employing a wheel (in the first case, a light aluminum wheel of 250 mm on ball bearings; in the second case, a bicycle wheel from which the tire had been removed) and a steel wire passing over it with a suitable weight attached.

When the car was at rest, the dynamometer itself had an error of less than 1 g; the smaller device with plane, up to 10 g; and the larger, up to 40 g. When subjected to vibrations, however, the inertia was much less.

3. Contemplated Determination of Form Resistance.

Unfortunately, it is not practically possible to tow a plane of infinitely small thickness. For the sake of strength and especially of rigidity, the thickness must be increased, as the length is increased and still more as the width is increased, especially for high speeds. Froude succeeded with a thickness of only 4.76 mm even for planes 15 m long and 480 mm wide, but only for speeds up to 3.03 m/s. For the new experiments with similar planes, however, the thickness was determined by the fact that a thickness of the wood of 2.5 mm was necessary for attaching the tapered ends to the smallest plane of 1.25 m length. This automatically required a thickness of 20 mm for the 10 m plane.

It also seemed inexpedient to increase the length of the taper to more than twenty times the thickness of the plane. For such an increase in the thickness and for the desired high speeds, it was considered no longer possible to disregard the wave-forming effect, as Froude did, or simply to substitute the resistance of a thin painted steel plane for that of the smooth tapered brass sections fitted together, as was done in the Dresden experiments.

In whatever way the subject was approached, it was impossible to determine accurately the effect of the thickness and of the tapering, so as to eliminate them. Although this was possible, under certain conditions, for the front end, it was impossible to determine accurately the displacement or form resistance at the rear end, which found itself in the water current produced by the whole of the plane preceding it, even by measuring the velocity of the water at this point.

In order, at least, to be able to introduce an approximate value for this resistance, the following method was adopted.* A body 1 m long was to be made by inserting a short lacquered wooden plane between the tapered ends of the 10 m plane; likewise between the ends of the 7.5 m and of the wider 5 m plane, hereinafter to be designated as the 20 mm, 15 mm, and 10 mm ends. Their resistances were to be determined for the contemplated range of speeds. Moreover, a lacquered brass plane, about 2 mm thick, sharply tapered at both ends, and of the same length as the combined ends attached to the wooden planes, was to be towed with a like submersion of 375 mm. The resistance of this brass plane was to be taken as the pure frictional resistance of the combined ends and the displacement resistance for the other tapered ends was to be calculated according to the law of similitude, although the similitude was only conditional. Since, however, the displacement resistance of the two ends of a plane, at least of the rear end, had to be smaller than the resistance thus determined, only 3/4 of the total was to be introduced into the calculation. The re-

* The air resistance for the apparatus could not be determined by special experiments. For the planes, it was mostly eliminated by the method employed.

sistance of the rear section of the plane would, therefore, come into the calculation only with the half of the combined ends, an arbitrary but not unreasonable assumption.

First, therefore, the three pairs of tapered ends and a correspondingly tapered brass plane were prepared. The perfect production of the latter was difficult and was accomplished only by repeated heating and careful hammering and finally by polishing with fine emery. It was then carefully lacquered and had a thickness of 2.3 mm in the finished condition.

The tapered ends for the smallest planes were made of solid brass, but for the larger planes of sheet brass, which was drawn over smooth iron wedges and fastened together with copper rivets (Fig. 3). The spaces between the wedges were filled with paraffin. They were joined to the planes with countersunk screws. All rough places were polished and lacquered. The lead keels were likewise attached with countersunk screws and then lacquered. The joints were polished, so that the planes presented a perfectly uniform, smooth surface.

4. Experiments with the Tapered End Sections and with Similar Planes in Cold Water.

On April 11, 1916, the experiments were begun, the brass plane being first attached to the towing apparatus. All the difficulties inherent in the towing process immediately appeared and increased rapidly with the speed. It is not easy to attach a plane with perfect accuracy, in the direction it is to be towed, by only one edge. Even the slightest error affects the measurements. The rails employed were accurately shaped, so as to follow a great circle of the earth within 0.1 mm, and all running surfaces were most accurately planed under exactly the same tension to which they were subsequently subjected on the walls of the basin. The greatest care was also exercised to construct the towing car so that it would be free from vibrations. Nevertheless the brass plane immediately began to vibrate transversely to the line of attachment and at a speed of 6 m/s it suddenly bent double. It was carefully straightened and remounted with still greater accuracy. On April 14 it was again ready for use. In the meanwhile the experiments with the joined pairs of tapered ends had been begun. Since vibrations also arose in the towing of the 20 mm and 10 mm end-sections, the experiments were carried to a speed of only 6 m/s, in order to avoid accidents. The 15 mm, on the contrary, could be towed at higher speeds, since the vibrations were less.

The brass plane twice more suffered the same accident at a speed of about 6 m/s, so that it was finally raised and towed with only 200 mm submerged.

In the intervals while the brass plane was being straightened, the towing experiments with the wooden planes were immediately begun, since all the time had to be utilized, in order to have as nearly uniform water temperature as possible for all the experiments, there being, at this season of the year, danger of its increasing rapidly. On May 1, the experiments were temporarily discontinued, all the planes, excepting the 10 m plane, having been towed. The latter was not finished and was first towed a year later (April 17, 1917) at a water temperature of about 10°C.

The results of these experiments are shown by Figs. 4,5 in the usual manner. Each test furnished one point in a system of coordinates, whose abscissa represents the m/s and whose ordinate represents the corresponding resistance. Through the points thus obtained, curves were subsequently drawn as accurately as possible. The dates are also given.

We will first consider Fig. 4. The curves are numbered in the order of the experiments. Curves 1, 4, 7, 8 and 9 represent the experiments with the brass plane with a subversion of 375 mm. Only curves 4 and 7 were completed, the other three being discontinued as unimportant and confusing. At the lower speeds these show a considerable divergence from each other. Curve 7 shows considerably larger resistance values than curve 4, which cannot be drawn smoothly through the individual points, since it first ascends slowly, then steeply and then, for higher speeds, forms a new slowly ascending curve. It is reasonable to assume that curve 4 first represents laminar friction, then a transition stage and then turbulent friction, while curve 7 represents no laminar stage at all.* The other curves apparently represent a mixed condition. At higher speeds, all curves converge into one curve of turbulent frictional resistance.

* At this point, it is appropriate to mention that, for ship models
(Continued at bottom of page 14.)

The scattering, which occurred in the experiments, can only be explained by an imperceptible tension of the planes, or by a changed condition of their surface from long immersion in water (opening of glued cracks or roughening of the lacquer).

The 10 mm pair of tapered end sections gave resistances, which nearly coincided with the above branching of the resistance curve of the brass plane. The 15 mm ends, on the contrary, gave a resistance curve, which at first coincided with the lower fork of the resistance curve for the brass plane, but subsequently branched upward. It is possible that a repetition of the towing experiments with both these pairs of end sections might give the other fork of the resistance curve of the brass plane, or intermediate curves.

For greater lengths, the branching would surely have ended at lower speeds, the same as for ship models, and it is safe to assume that, for planes as well as for tubes, a certain length is required to produce the condition of complete turbulence. For tubes, Blasius required about fifty times the diameter for the starting distance, the location of the same being dependent on the

*(Contd. from page 13)
two entirely separate resistance curves are obtained. There is no scattering of the resulting points, which fall either in the upper or lower resistance curve. For illustration, Fig. 4a is added here. How it happens, that in one instance the upper and in the other instance the lower branch is followed, has not yet been explained. No means has yet been found to compel one or the other, for the individual results often follow one another on one day in the one branch and on the next day in the other branch. The experimental apparatus in the Vienna Institute is so perfect, that a resistance measurement seldom falls outside of the curve subsequently drawn through the individual measuring points. The branching took place only for speeds of 1.1-1.4 m/s and principally with the 5 m model. On the other hand, the least temperature change in the water had the expected effect on the results.

speed. Even then we must observe a certain product of the length times the speed, whose magnitude seems to be about 5 m^2/sec. For ship models, we do not have to calculate the whole length of the surface, but only to the point where the frictional boundary layer of water separates.

There is nothing special to note regarding 20 mm pair of end sections.

This is not the case with the results of the experiments with the wooden planes plotted in Fig. 5, aside from the otherwise quite unwonted scattering of the measuring values, in which a reason for the difficulty of such experiments is obvious. The absence of such a noticeable scattering in all the Froude and Dresden diagrams is partially due to the fact that substantially lower speeds were employed and also to the fact that the planes were apparently tested only a few times and always on the same day. This would explain why the day-effect, now exhibited, either because of a change in the surface or in the tension, was not noticed in the earlier experiments. It might be advisable, in the future, never to leave the planes in the water more than one working day and to apply a new coat of lacquer before each experiment.

5. Magnitude of Form Resistance.

We must now try to make, from the irregular measurements of the resistances of the brass plane and of the pairs of tapered end sections for the displacement resistance of the planes, the deductions necessitated by the resistances of the planes, in order to determine the surface resistance. For this purpose, we will follow the course already adopted as the basis for the experiments in question.

To begin with the determination of the magnitude of the resistance of the brass plane, the smaller submersion depth gave greater stability. Table II gives the values obtained. It is seen that these values conform very well to a quadratic speed law. From them the values for the more deeply submerged planes were calculated, corresponding to the increase in area of the submerged surfaces, which, with the observed values for the upper branch of the fork, are also given in Table II. These two values agree very well, so that they may be regarded as satisfactory for most cases of turbulent friction. Here also the values would conform to a quadratic speed law, as shown in column e. In order to correspond to the experiments with 200 and 375 mm submersion, the values in column d were determined by graphic means and regarded as surface resistance.

Table II

a	b	c	d	e	f
Speed	Resistance	Resistance calculated from plane 1	Resistance found from b and c	Resistance according to quadratic law	Remarks
m/s	g	g	g	g	
	Brass plane I, lacquered, 1 m long, 200 mm submerged. Temperature of water, 10.7°C				
1	73[a)]			63	a) Very variable
2	238[a)]			250	
3	551			563	
4	1001			1000	
5	1552			1562	
6	2225			2250	
7	3036			3060	
8	4000			4000	
(7.5)	(3502)			(3520)	
	Brass plane II, lacquered, 1 m long, 375 mm submerged. Temperature of water 9.7°C.				
1	106[b)]	137	106	117	b) Read from upper branch of fork
2	449	446	448	470	
3	1112	1033	1090	1053	
4	1968	1876	1935	1875	
5	2955	2920	2940	2940	
6	4165	4170	4170	4222	
7		5690	5690	5750	
8		7500	7500	7500	
(7.5)		(6570)	(6570)	(6600)	

The determination of the form resistance for the tapered end sections seems much less reliable. The results of the resistance measurements are given in Table III, column b. By subtracting therefrom the surface resistance, we obtain the form resistance given in column c. This cannot be negative, however, and it is evident that here a more or less laminar friction has created such disorder. Thereby perhaps, with the thinner end sections, the longer middle sections produced an especial effect. But how shall we determine this effect from the available data and what values shall we finally adopt for the similar long planes?

The value of 600 g was adopted as pure form resistance for the speed of 6 m/s for the 20 mm end sections and it was assumed that this corresponded to the square of the speed. It would accordingly give column d of Table III.

It was further assumed that the form resistance follows Froude's law of similitude and that it accordingly increases with the third power of the ratio of similitude for speeds which vary as the square root of the ratio of similitude and for similar planes. Thus column c of Table IV was obtained. Lastly, the already-mentioned consideration was accepted and three-fourths of the value of the thus-determined form resistance was subtracted from the resistances of the towed planes and the remainder was then regarded as the surface resistance.

Table III.

a	b	c	d
Speed	Resistance	Differs from plane resistance	Displacement resistance according to quadratic law
m/sec.	g	g	g
Pair of ends, 10 mm thick, 1 m long, 375 mm submerged. Temperature of water 10.5°C			
1	46	13	
2	461	22	
3	1112	-[illegible]2	
4	1933	18	
5	2958	80	
6	4250	155	
Pair of ends, 15 mm thick, 1 m long, 375 mm submerged. Temperature of water 9.7°C			
1	95	- 11	9
2	322	-126	36
3	881	-209	80
4	1845	- 90	142
5	3040	-100	222
6	4490	320	320
7	6095	405	435
(7.5)	(7000)	(430)	(500)
Pair of ends, 20 mm thick, 1 m long, 375 mm submerged. Temperature of water 10.2°C			
1	130	24	17
2	573	125	67
3	1300	210	150
4	2225	290	267
5	3400	460	420
6	4770	600	600

As shown by Table IV, no important results were obtained from testing the end sections and comparing their resistances with that of a thin lacquered brass plane, since the form resistance, obtained chiefly by all sorts of considerations and but little by direct measurement, averages less than 1% of the total resistance, so that, even without its subtraction, the surface resistance, at least of the small planes, would seem to be obtained with sufficient accuracy by the simple towing experiment with the whole plane. Only with the large planes does the form resistance (up to 1.5% with the 10 m plane) finally become noticeable.

The results were further evaluated, immediately after the determination of the surface resistance, by plotting them in Fig. 9. Hence as early as May, 1916, the correct principles were established, which were corroborated by all subsequent experiments.

Before considering this matter further, however, it seems advisable to become acquainted with the progress of the experiments and the values obtained and, in conclusion, to give a summary of them all and of the laws established by them.

Table IV.

Resistances of lacquered wooden planes, all similar, with lacquered brass end sections, at low water temperature.

a	b	c	d	e
Speed	Measured resistance of planes and end sections	Displacement resistance of end sections alone	Displacement resistance of end sections in combination with the planes	Surface resistance of planes
m/s	g	g	g	g
	Plane 1.25 m long; temperature of water 10.2°C			
1	15	0.35	0	15
2	120	1.4	1	120
3	280	3.2	2	280
4	480	5.6	4	480
5	750	8.7	7	740
6	1035	12.5	9	1025
7	1380	17	13	1365
8	1770	22.4	17	1750
(7.5)	(1570)	(19.5)	(15)	(1550)
	Plane 2.5 m long; temperature of water 9.9°C			
1	110	1.39	1	110
2	410	5.56	4	410
3	880	12.5	9	870
4	1550	22.2	17	1530
5	2330	34.7	26	2290
6	3220	50	37	3170
7	4400	68	51	4340
8	5840	89	67	5770
(7.5)	(5120)	(78.2)	(59)	(5050)
	Plane 5 m long; temperature of water 9.7°C			
1	400	5.56	4	400
2	1560	22.2	17	1540
3	3310	50	37	3270
4	5700	89	67	5620
5	8620	139	104	8520
6	12040	200	150	11890
7	16200	272	204	16000
8	21180	356	267	20910
(7.5)	(18590)	(313)	(235)	(18350)

Table IV (Cont.)

Resistances of lacquered wooden planes, all similar, with lacquered brass end sections, at low water temperature.

a	b	c	d	e	f
Speed m/s	Measured resistance of planes and end sections g	Displacement resistance of end sections alone g	Displacement resistance of end sections in combination with the planes g	Surface resistance of planes g	
	Plane 7.5 m long; temperature of water 10.7°C				
1	840	12.48	9	830	
2	3240	49.9	37	3200	
3	6860	121.4	91	6770	
4	11790	200	150	11640	
5	18010	312	236	17770	
6	25320	450	337	24980	
7	34000	612	459	33540	
8	44000	799	599	43400	
(7.5)	(38860)	(703)	(527)	(38330)	
	Plane 10 m long; temperature of water 8.3°C				
1	1450	22	17	1430*	*400 g at 0.5 m/s
2	5670	89	67	5600	
3	12360	200	150	12210	
4	21240	356	267	20970	
5	31780	556	417	31360	
6	44320	800	600	43720	
7	59600	1090	817	58780	
8	---	1420	1315	---	
(7.5)	(68300)	(1252)	(940)	(67360)	

6. The Wide 5-Meter Plane and the 5-Meter Plane at Different Degrees of Submersion.

We have already referred to the construction of a 5 m plane and of a similar plane with twice the submersion depth, to be used for the similitude experiments, and briefly indicated the reason therefor, which we will now explain more fully.

If we consider a plane, of infinitely small thickness in front, standing vertically in water with its upper edge projecting, it is evident that the upper portions of the surface, when the plane is moving horizontally, affect only the lateral layers of water and, since all portions of the surface have the same speed, we may, perhaps, assume (a sufficient width of the plane being taken for granted) that every upper particle of the surface acts on a water prism perpendicular to the surface.

But, on the lower edge of the plane, the water particles can pass from one side to the other and, since the layer of moving water acquires thickness with increasing length, it is obvious that under the lower edge of the plane there is also a layer of water of corresponding thickness, which must be carried along by the lower portion of the plane.

This absorbs power and the logical conclusion is that narrower planes must have a greater specific surface resistance than wider planes. It is therefore desirable, for the determination of the general law of surface resistance of rectangular planes, to try towing experiments not only with planes of various lengths, but also of the same length but different widths.

The wider 5-meter plane was towed on April 25 and 26, 1916, along with the first experiments with similar planes. The results are therefore included in Fig. 5. The resistance points, at the highest speeds, showed considerable scattering, probably due to disturbing vibrations. It is indeed conceivable that such a wide plane of such thinness is more flexible than a thicker one and that a greater length of the flexible body communicates its vibrations more readily to the holding device, however rigid the latter may be. It was therefore endeavored to draw the curves through the points of minimum resistance, especially for the highest speeds.

Table V.

a	b	c	d	e
Speed m/s	Resistance of plane with end sections g	Displacement resistance of end sections combined with plane g	Surface resistance (in round numbers) g	Surface resistance calculated from that of the plane submerged 250 mm g
Plane 5 m long, 500 mm submerged; water temperature 10.2°C				
1	780	8	770	784
2	2950	34	2920	3050
3	6360	74	6290	6480
4	10980	134	10850	11130
5	16800	208	16590	16870
6	23700	300	23400	23550
7	31910	408	31500	31700
8	41940	534	41010	41400
(7.5)	(36680)	(470)	(36210)	(36150)

Table V contains the numerical values of the measured resistance, that found for the form resistance and, in column d, the difference between the former two, as the surface resistance at various speeds. For comparison, the proportionate resistance of the 5 m plane, submerged 250 mm, is given in column e. This is calculated by the formula $w \times \frac{1.01}{0.51}$, in which w denotes the surface resistance of the first plane. As here shown, the specific surface resistance of the wider plane (column d) is smaller than that of the narrower plane (column e), notwithstanding the possibility of greater vibrations.

This result emphasized the need of further experimentation with narrower and narrower submerged surfaces, by letting the 5 m plane project farther and farther above the water. It was not until April, 1917, that time was found to perform these experiments, the average temperature of the water then being 8°C.

First, the 5 m plane was again towed with a submersion of 250 mm, in order to connect with the previous experiments and also as a basis of comparison. It was then towed at submersion depths of 150, 100, 50 and 25 mm, the results being shown in Fig. 6. It immediately became evident that the resistance was not proportional to the submersion, but increased more slowly than the depth of submersion. In order to keep on the safe side and not incur the risk of having to hunt for a cause in the lead keel and its method of attachment, another plane without any lead keel was quickly constructed and towed in the same way. As shown by Fig. 3, the results agree throughout with those obtained with the former plane.

The objection that greater vibrations increased the resistance of the lower portion of the surface cannot be denied, but nevertheless the smaller specific resistance of the twice-as-wide plane was obtained. The numerical values of these experimental results and the surface resistance obtained therefrom, in a manner similar to that previously employed, are contained in Table VI. The displacement resistance was considered as proportional to the submersion depth and was taken from Table IV.

If we compare the surface resistance, now found for the 5 m plane and a submersion depth of 250 mm, with that previously obtained, we observe that the later results are throughout somewhat greater than the earlier. This is partially due to the lower temperature of the water. The balance may be reasonably regarded as coming within the limits of the accuracy attainable in experiments of this kind.

The further evaluation of the experimental results will be made later, in connection with all the others.

7. Experiments in Warm Water.

It is comprehensible that an endeavor should be made to determine the effect of the water temperature on the experiments with planes, for the sake of completeness. This experiment is easily performed with tubes, since it is relatively easy, without great expense or troublesome devices, to heat the requisite amount of water to quite a high temperature. It is otherwise in experiments with planes on the scale under consideration. The artificial heating of the 8000 to 9000 m^3 of water in the basin was not practicable. It was only practicable to utilize the natural summer increase in temperature. It was extremely doubtful, however, as to whether the few degrees difference would give sufficiently accurate results, due to the difficulty of performing really perfect experiments with regard to so many other factors.

Table VI.

Resistance of a lacquered plane, 5 m long and 10 mm thick, towed at different submersion depths in water with an average temperature of 8°C. Experiments performed in April, 1917.

a	b	c	d
Speed m/s	Measured resistance of planes with tapered end sections g	Displacement resistance g	Surface resistance (in round numbers) g
	Submersion depth 25 mm		
1	50	0.4	50
2	220	1.7	220
3	490	3.7	490
4	840	6.7	830
5	1200	10.4	1190
6	1610	15	1595
7	2180	20.4	2160
(7.5)	(2550)	(23.5)	(2525)
	Submersion depth 50 mm		
1	90	0.8	90
2	360	3.4	360
3	785	7.4	780
4	1370	13.4	1355
5	2085	20.8	2065
6	2935	30	2905
7	3950	40.8	3910
(7.5)	(4500)	(47)	(4450)
	Submersion depth 100 mm		
1	160	1.6	160
2	630	6.8	625
3	1405	14.8	1390
4	2480	26.8	2455
5	3750	41.6	3710
6	5240	60	5180
7	7030	81.6	6950
(7.5)	(8060)	(94)	(7965)

Table VI (Cont.)

Resistance of a lacquered plane, 5 m long and 10 mm thick, towed at different submersion depths in water with an average temperature of 8°C. Experiments performed in April, 1917.

a	b	c	d
Speed m/s	Measured resistance of planes with tapered end sections g	Displacement resistance g	Surface resistance (in round numbers) g
	Submersion depth 150 mm		
1	244	2.4	240
2	995	10.2	985
3	2105	22.2	2080
4	3565	40.2	3525
5	5390	62.4	5330
6	7560	90	7470
7	10250	122.4	10125
(7.5)	(11850)	(141)	(11710)
	Submersion depth 250 mm		
1	400	4	395
2	1570	17	1555
3	3330	37	3290
4	5730	67	5660
5	8700	104	8595
6	12340	150	12190
7	16630	204	16425
(7.5)	(18980)	(235)	(18745)

It was nevertheless decided to try the experiment and it was hoped at least to verify the values previously obtained, even though the results might not be accurate enough to determine the effect of the heat. Unfortunately, it was impossible to restore the more or less scratched planes to their original perfect condition, which we would have been glad to do. Time was lacking and shellac had become extremely rare, owing to the blockade, so that we could not lacquer the planes as a whole, but only the roughest places. The experiments were performed in the period from August 24 to 29, 1916. Not much time could be devoted to them and consequently not all the planes could be used. In order, however, to obtain as comprehensive results as possible, the brass plane and the pairs of tapered end sections were again used in the experiments.

The results of the individual experiments are shown in Fig. 7. No scattering was observed, with the exception of two results of the experiments with the 7.5 m planes. Tables VII and VIII give the numerical values of the curves drawn through the individual measuring points.

Table VII.

a	b	c	d
Speed (m/s)	Measured resistance g	Calculated resistance for 375 mm submersion depth g	Difference from resistance of brass plane g
Lacquered brass plane, 1 m long, 2.3 mm thick, 200 mm submerged. Temperature of water 18.5°C			
1	75	140	
2	250	470	
3	570	1070	
4	1040	1950	
5	1640	3070	
6	2340	4380	
7	3140	5880	
(7.5)			

Table VII (Cont.)

a	b	c	d
Speed (m/s)	Measured resistance g	Calculated resistance for 375 mm submersion depth g	Difference from resistance of brass plane g
	Pair of end sections, 10 mm thick, 375 mm submerged. Temperature of water 18.7°C		
1	140*		0
2	560		90
3	1250		180
4	2110		160
5	3170		100
6	4400		20
7	5870		-10
(7.5)			
	Pair of end sections, 15 mm thick, 375 mm submerged. Temperature of water 18.7°C		
1	140**		0
2	560		90
3	1220		150
4	2090		140
5	3200		130
6	4530		150
7	6080		200
(7.5)			
	Pair of end sections, 20 mm thick, 375 mm submerged. Temperature of water 18.8°C		
1	160**		20
2	630		160
3	1320		250
4	2190		240
5	3310		240
6	4700		320
7	6300		420
(7.5)			

* Greater than previously found.

** Found greater below, the same above, as compared with previous measurements. End sections and planes were both probably somewhat scratched.

By comparing Tables II and VII, we find that the resistance of the brass plane was somewhat greater in the warmer water than it was previously in the colder water. This inconsistent result was probably due to the somewhat imperfect condition of the surfaces in the later experiments. The same is the case with the two pairs of smaller end sections (See Tables II and VI). Only the pair of end sections 20 mm thick gave, in part, smaller results than previously.

Even if the results of these summer experiments serve no other purpose, they constitute a valuable confirmation of the winter experiments and are repeated here as such and as a proof of the need of absolutely perfect surfaces and of the difficulty of obtaining reliable data from such experiments.

Table VIII.

Resistance of similar planes and of a wider plane in warmer water.

a	b	c	d	e	f
Speed m/s	Measured resistance of planes with tapered end sections g	Surface resistance (in round numbers) g	Difference from Table IV, e g	Difference % of previously measured surface resistance %	Surface resistance at 500 mm submersion calculated from 250 mm submersion g
	Plane 2.5 m long, 5 mm thick, 125 mm submerged. Temperature of water 18.8°C				
1	110	110	0	0	
2	410	410	0	0	
3	880	870	0	0	
4	1540	1520	-10	-0.6	
5	2350	2380	+30	+1.3	
6	3230	3190	+20	+0.6	
7	4480	4430	+90	+0.42	
(7.5)	(5100)	(5030)	(-20)	(-0.4)	

Table VIII (Cont.)

Resistance of similar planes and of a wider plane in warmer water.

a	b	c	d	e	f
Speed	Measured resistance of planes with tapered end sections	Surface resistance (in round numbers)	Difference from Table IV,e	Difference of previously measured surface resistance	Surface resistance at 500 mm submersion calculated from 250 mm submersion
m/s	g	g	g	%	g
	Plane 5 m long, 10 mm thick, 250 mm submerged. Temperature of water 18.8°C				
1	430	430	+ 30	+7	
2	1570	1550	+ 10	+0.6	
3	3300	3250	- 20	-0.6	
4	5620	5550	- 70	-1.3	
5	8450	8350	-170	-2.1	
6	11920	11750	-140	-1.3	
7	16240	16040	+ 40	+0.25	
(7.5)	(18700)	(18470)	(+120)	(+0.65)	
	Plane 5 m long, 10 mm thick, 500 mm submerged. Temperature of water 18.5°C				
1	800	790	- 20	-2.5	852
2	2940	2910	- 10	-0.35	3070
3	6230	6160	-130	-2.1	6440
4	10770	10640	-210	-3.4	11000
5	16460	16250	-340	-2.1	16550
6	23240	22940	-460	-1.9	23300
7	31360	30950	-550	-1.8	31800
(7.5)	(35760)	(35290)	(-920)	(-2.5)	(36600)
	Plane 7.5 m long, 15 mm thick, 375 mm submerged. Temperature of water 18.4°C				
1	890	880	+ 50	+5.7	
2	3210	3170	- 30	-1	
3	6790	6640	± 0	±0	
4	11680	11430	-120	-1	
5	17950	17710	- 60	-0.3	
6	25420	25080	+100	+0.4	
7	33960	33500	- 40	-0.12	
(7.5)	(38500)	(37970)	(-360)	(-0.9)	

The experiments with the wooden planes having tapered brass end sections came out better (See Tables VI and VIII). The experiments with planes in warmer water gave, throughout, the anticipated smaller resistances. The difference seems too small, but, when we consider that the end sections alone in warmer water, as already demonstrated, gave greater values than previously in the colder water, then these results must be regarded as very satisfactory, in view of the experimental conditions.

In this connection it may be opportune to call attention to the fact that the lacquer separates easier from the metal than from the wood, so that, with repeated use, the end sections developed relatively more roughness than the wooden planes between them, especially as they were used more. This affords a simple explanation for the apparent inconsistencies and enables us to assume a greater resistance difference for a uniform surface condition than that obtained for the temperature difference of about 8.5°C.

As the deduction for the displacement resistance, the same values as before were employed. The differences thus obtained in absolute and percentage values are given in Table VIII, columns d and e. The best values were apparently given by the 5 m plane with 500 mm submersion.

If we again compare them with the new values of the narrower plane (Table VIII, column f), we again find that the wider plane shows a smaller specific resistance than the narrower plane and we can perceive in this fact a new proof of a special resistance generated by the lower longitudinal edge.

The values for the low speeds must be employed in all cases with discretion, since any experimental errors are expressed the most strongly as percentages.

8. Renewed Experiments with the 10-Meter Plane.

Previously the 10 m plane had to be towed with apparatus designed for smaller stresses. Strong springs had to be added, in order to measure the great resistances and, moreover, the support was too weak for the strong oscillations of the heavy plane. The evaluation of the results afforded cause for fearing that the experiments had, throughout, given too large values. Hence it was planned to repeat the experiments with the 10 m plane, after finishing the strong apparatus designed for towing planes up to 20 m in length. Unfortunately, other demands on the Institute delayed the execution of these plans until March 1, 1918.

Table IX.

a	b	c	d
Speed m/s	Resistance of plane with end sections g	Displacement resistance of end sections in combination with the planes g	Surface resistance (round numbers) g
	Plane 10 m long, 500 mm submerged in water at 7.1°C		
1	1450	17	1430
2	5300	67	5230
3	11700	150	11550
4	20600	267	20330
5	31200	417	30780
6	43300	600	42700
7	57850	817	57030
(7.5)	(66300)	(964)	(65330)

Table X.

λ_p values for similar surfaces and for a dissimilar brass plane.

a	b	c	d	e
Length of plane m	Resistance at speed of 1 m/s* g	Area in m^2	 λp	Temperature of water in degrees C
1.25	0.034	0.1593	0.214	10.2
2.5	0.109	0.637	0.171	9.9
5	0.405	2.548	0.159	9.7
7.5	0.865	5.735	0.151	10.7
10	1.48	10.192	0.145	8.3
1 Brass plane	0.078	0.402	0.195	10.7

* Read from Fig. 9 (in kg).

Table XI.

Dresden experiments in 1908.
Resistance of planes of various lengths, 8 mm thick, 768 mm perimeter, coated with dull ground lacquer.

a	b	c	d	e
Speed in m/s	l =1.6 m	l =3.6 m	l =4.6 m	l=6.5 m
1	0.2	0.45	0.56	0.75
2	0.8	1.63	2.07	2.80
3	1.75	3.5	4.36	5.95
4	3.05	5.81	7.37	10.12
(4.5)	--	--	--	(12.65)
5	4.6	9.23	11.37	--
Resistance at 1 m/s speed, when it increases with the 1.875 power of the same (calculated).	0.220	0.445	0.550	0.750
λp	0.179	0.163	0.156	0.150

The experiments, in fact, gave lower values, notwithstanding the lower temperature of the water (See Fig. 8 and Table IX). No time remained to investigate the scattering of the results between 5.4 and 6 m/s.

9. Further Evaluation of the First Experimental Results.

Fig. 9 gives the results of the experiments with the planes. The abscissas and ordinates are both divided logarithmically (the purchasable logarithmic paper causes the zero point to be located on the right-hand side). The resistances for each whole and in part for each half meter of speed were first plotted on the left-hand side from the curves obtained in the resistance experiments. As already mentioned, the diagram was only produced gradually, the results of Froude's experiments in 1872 and of the Dresden experiments being also incorporated.

Froude's results were taken directly from the curves given by him for the coating with Hays Composition and recalculated for meters and kilograms (Table XII). This coating gave, throughout, the smallest values and must therefore be regarded as the smoothest smoother even than the lacquer, also used, which gave a slightly smaller resistance for only the shortest plane. Table XI gives the values of the Dresden experiments for ground lacquer.

In comparing the resistances in the logarithmic diagram, we immediately note that all the resistance lines are straight and have exactly the same slope. The only exception is the narrowest 5 m plane in its central section, but at higher speeds, it also follows exactly the same law as the others. In view of what has already been said concerning the combination of turbulent and laminar friction, it cannot be considered strange that, for lower speeds and shorter planes, the resistance should lie below the corresponding straight line. It can, however, be confidently asserted that all resistances, due to a turbulent condition, conform to the same law of potential speed, regardless of whether the planes are long or short, wide or narrow. This law is followed not only by the 5 m plane in all its degrees of submersion, but also by Froude's planes and the Dresden planes with as great a degree of accuracy as can be expected in this sort of experiments. We can not judge as to how carefully Froude's experiments were executed, but the Dresden experiments were performed without haste and with the greatest care by the writer himself. It is self-evident that the plane 60 cm long will not conform, if it is remembered that approximately $v\,l = 5\ m^2/sec.$ is the lower limit of the purely turbulent condition.

It is now clear as to how it happened that both Froude and the writer earlier found decreasing powers of the speed and adopted them as the basis of their resistance formulas. It was simply because the more or less laminar condition was considered. But the recent investigation, which considerably raised the attainable speed limit, establishes with the greatest certainty yet attained (excepting for low speeds and short planes with $v\,l \leq 5\ m^2/sec.$), the law that <u>the resistance values of turbulent friction, for square-edged planes with smooth surfaces, increase, for every ratio of the length to the width with unappreciable thickness, exactly as the 1.875 power of the speed</u>.

The following symbols will now be introduced:

l = length of plane in meters;

b = submerged width of plane in meters;

u = submerged vertical perimeter of plane in meters;

d = thickness of plane in meters;

W = measured resistance of planes and tapered ends in kilograms;

w_v = displacement or form resistance in kg;

w = surface resistance in kg;

w_k = resistance of longitudinal edge in kg;

$w_p = w + w_k$, when thickness = zero;

w_{sp} = specific surface resistance at any point in kg/m²;

v = speed in m/sec.;

λ = coefficient of turbulent friction for pure surface resistance;

λ_p = coefficient of friction for infinitely thin planes of given shape;

γ = density of liquid in kg/m³;

$\rho = \frac{\gamma}{g}$ = density in $\frac{\text{kg sec.}^2}{\text{m}^4}$;

$\nu = \frac{\text{technical viscosity coefficient}}{\text{density}} = \frac{\eta}{\rho}$ = kinetic friction in m²/sec.

The discovered law is therefore:

$$w_p = v^{1.875}\ c$$

in which c is a constant peculiar to the plane under consideration.

Table XII.

Froude's Experiments, 1872.

Resistance of planes of various lengths, 4.762 mm thick, 964.78 mm submerged perimeter, coated with "Hays Composition" (which gave the least resistance).

a	b	c	d	
Speed m/s	Feet per minute	Resistance of air(lb.)	Plane I, length 1.524 m lb.	kg
1	196.854	0.05	0.61	0.277
2	393.708	0.12	2.51	1.139
3	590.562	0.18	6.0	2.72
(3.5)	688.979	0.21	8.07	3.66
4	797.416	0.24	10.36	4.7
Resistance at a speed of 1 m/s when it increases with the 1.875 power of the same.				0.345
λ_p				0.235

	e		f		g	
	Plane II, length 4.877 m lb.	kg	Plane III, length 8.534 m lb.	kg	Plane IV, length 15.24 m lb.	kg
	1.75	0.794	3.25	1.475	5.1	2.31
	6.58	2.985	11.40	5.17	17.98	8.16
	14.17	6.43	23.97	10.87	38.37	17.40
	19.0	8.62	31.7	14.38	50.7	23.00
	24.21	10.98	40.09	18.18		
Resistance at a speed of 1 m/s when it increases with the 1.875 power of the same.		0.81		1.34		2.16
λ_p		0.172		0.163		0.147

Table XIII.

W. Froude's Speed Experiments.

a	b	c
Length of plane (m)	Lacquer	Paraffin
0.61	2.0	1.95
2,44	1.85	1.94
6.10	1.85	1.93
15.24	1.83	

While William Froude considered the exponents diminishing with the length as correct (Table XIII), his son, R. E. Froude (Transactions of the Inst. of N.A., 1888, R.E., Froude "On the Constant System" etc.), on the basis of his father's and his own experiments, believed in the adoption of the exponent 1.825 for all lengths. The Dresden experiments had, however, already shown this exponent to be too small. At that time the resistance curves, calculated with these exponents, were introduced into the diagram and it can be seen that the measured resistances in the lower portion of the curves are smaller, but, in the upper portion, gradually grow larger, than the computed curve would indicate. The reason R.E. Froude calculated his exponents too small, lies probably in the low speeds, which led to a greater allowance for the laminar friction.

This greater exponent now appears to be confirmed, even by the results of earlier experiments.

For motion phenomena in liquids, in which the viscosity plays the deciding role, Osborne Reynolds adopted an especially favorable form of the law of similitude. (Reynolds, "Phil. Transactions of the Royal Society of London, Vol. 174, 1883, pp. 935 an 273.)

This law has been derived by Blasius ("Mitteilungen über Forschungsarbeiten auf dem Gebiete des Ingenieurwesens" No. 131, 1913, p. 5) and more recently by Weber (Jahrbuch der Schiffbautechnischen Gesellschaft, 1919). The latter presented the entire subject in such a clear and comprehensive manner in his lecture "Die Grundlagen der Aehnlichkeitsmechanik und ihre Verwertung bei Modellversuchen unter besonderer Berücksichtigung schiffbautechnischer Anforderungen" before the "Schiffbautechnische Gesellschaft" in March, 1919, that it is here better simply to refer to it, than to make an abstract of it. The essential points for the present research can be more readily gathered from Weber's article.

An important form of Reynolds law of similitude reads: "If two motion phenomena take place in a mechanically similar manner in noncompressible fluids under the sole action of viscosity, then the corresponding expressions $\frac{(v)(l)}{\nu}$ and $\frac{vl}{\nu}$ give the same Reynolds number ψ. As a nondimensional coefficient, ψ is independent of the mass units employed, so that its calculation (always on the assumption of mechanically similar phenomena) gives the same value in the different systems. Under these conditions, every two corresponding forces are directly proportional to the squares of the viscosity coefficients and inversely proportional to the density of the two fluids. If the fluids are alike in these respects, the forces exerted on the full-sized object and on the model are of equal magnitude.

"If k and K represent the given forces, then

$$\frac{k}{K} = \frac{\eta^2 (\rho)}{(\eta)^2 \rho} = \frac{\frac{\eta^2}{\rho}}{\frac{(\eta)^2}{(\rho)}} = \frac{\rho \quad \nu^2}{(\rho) \quad (\nu)^2} \tag{1}$$

or, on introducing the characteristic ζ,

$$k = \zeta \rho \nu^2 , \quad K = \zeta (\rho) (\nu)^2 \tag{2}$$

$$\zeta = \frac{k}{\rho \quad \nu^2} = \frac{K}{(\rho) \quad (\nu)^2} \tag{3}$$

Hence

$$\zeta = f (\psi) = f \left(\frac{v l}{\nu}\right) \tag{4}$$

When plotted in a rectangular system of coordinates, it gives the curve of the characteristic ζ.

"Each model experiment yields a definite Reynolds number $\psi = \frac{vl}{\nu}$ as abscissa and a definite characteristic $\zeta = \frac{k}{\rho \quad \nu^2}$ (likewise a pure number) as ordinate, a pair of values which are unchangeable for all mechanically similar phenomena and hence also for the principal phenomenon.

"We can also express the model force in the form of the general law of similitude ($k = \alpha \rho F v^2$) and thus obtain α as a function of ψ."

From this presentation of Reynolds law of similitude, it follows that, for the case when the resistance of a surface follows a potential law of the speed, the same potential law applies for the ratio l/ν, when $\zeta = f\psi$. The powers, however, must be the same as for v, so that the resultant expression will also be nondimensional.

If we should then succeed in determining what potential law of the length the resistances of similar surfaces follow for different lengths but the same speed, we would then come very near the potential law for ν.

Hence, in Fig. 9, the resistances found for similar planes are plotted on the right side against the lengths as abscissas for various speeds and lines are drawn through the points of equal speed. Thus we find that the same potential law, as for the speeds, also applies to the lengths, since all lines are parallel to those previously obtained, where w_d was plotted as a function of the speed. Only the resistances of the smallest planes (1.25 m long) did not agree, but were too large. The second law would accordingly read: "The resistances of similar smooth surfaces increase, for the same speed, as the 1.875 power of the lengths of the surfaces."

Hence it seems to be demonstrated that, at least for all other planes, the Reynolds law applies partially in its one case, since it was found that $\zeta = \alpha\,(v\,l)^{1.875}$. (5)

The important fact that the resistance of similar planes, having the same speed, varies with $l^{1.875}$, which we can confidently assume to have been demonstrated by the experiments, leads to a series of further general conclusions regarding the resistance of planes.

The old Froude formula would now read

$$w_p = \lambda_p \, F v^{1.875} \tag{6}$$

in which λ_p* is independent of the length, so that

$$\lambda_p = f\,(l) \tag{7}$$

If λ_p (designated by $\lambda_{p(1)}$ for 1 m length of plane and 1 m/s speed) were known, we could calculate the resistance w_p for all lengths and speeds for similar planes like those used in the experiments. In all cases, a pure turbulent condition must naturally be adopted as the basis. We would then have

$$w_p = \lambda_{p(1)}{}^{1.875} \times v^{1.875} \times F \tag{8}$$

On the assumption that the surface resistance is proportional to the width, it is possible for us to calculate from $\lambda_{p(1)}$ the resistance λ_p per unit area of variously shaped planes. If w_1 represents the resistance of a surface of $l = 1$ for $v = 1$ and f the submerged surface, we then have

$$\lambda_{p(1)} = \frac{w_1}{f} = \frac{w_1}{u\,l} \tag{9a}$$

and

$$\lambda_p = \frac{w_1}{f} \cdot \frac{l^{1.875}}{l^2} = \lambda_{p(1)} \; l^{-0.125} \tag{9}$$

Fig. 9 shows that, in fact, the calculated λ_p values for

* λ_p = coefficient for vanishingly thin planes of the given form.

various plane lengths conform to this law. As the resistance for the speed of 1 m/s, there was occasionally adopted the intersection point of the line, plotted against the speed as the abscissa, with the 1 m ordinate. This resistance was divided by the mass of the given submerged surface in m² (Table X). The values previously found in Dresden, when computed with 1.875 as the power of the speed, conform well with the law for the variation of λ_p. We only need to assume that the temperature of the water in Dresden was somewhat higher than in the recent experiments (Table XI), but it is hardly possible to reconcile therewith the considerable upward deviation of Froude's values (Table XII). Here the planes must have been rougher or there must have been some other disturbing factor.

It is also obvious that the resistance of the 1.25 m plane will not conform to the λ_p curve, because it does not even conform to the power curves of l. The one-meter long brass plane, however, gave a satisfactory result.

The numerical value of λ_p may be easily found from the corresponding lines of Fig. 9, giving

$$\lambda_p = 0.195 \text{ (about)} \tag{10}$$

Froude's complicated numerical table of resistances with relation to the length are no longer tenable. The resistance of a smooth surface of any size, for any speed, may now be found according to Froude's formula, with omission of the specific gravity of the water, as follows

$$w_p = \lambda_p \, l^{-0.125} \, F \, v^{1.875} = 0.195 \, l^{-0.125} \, F v^{1.875} \quad (11$$

The effect of the temperature of the water is omitted in Froude's formula. According to our experiments, the new values would be correct only for about 10°C.

From the equation just found for surface resistance, we can also determine the specific resistance at any point and at any distance from the leading edge. If we call this distance L and calculate the resistance curve for different lengths for a width of one meter and for one and the same speed, then the tangent at any point of this resistance curve gives the specific surface resistance at the corresponding distance from the leading edge. (See Gümbel, "Das Problem des Oberflächenwiderstandes," p. 474 in the "Jahrbuch der Schiffbautechnischen Gesellschaft," Vol. 14, 1912.)

$$w_{p(\text{specific})} = \frac{d \int w_p \, d \, L}{d \, L} \quad (12)$$

Now, if we call $v = 1$, we have

$$\int w_p \, d \, L = \lambda_p \, L^{-0.125} \times 1 \;\; L = \lambda_p \, L^{0.875}$$

and hence $w_{p(\text{specific})} = \lambda_p \;\; 0.875 \, L^{-0.125}$ for $v = 1$ m (13)

or, for any given speed v,

$$w_{p(\text{specific})} = \lambda_p \, v^{1.875} \times 0.875 \, L^{-0.125}$$

$$w_{p(\text{specific})} = 0.195 \times 0.875 \, v^{1.875} \, L^{-0.125} \quad (14a)$$

$$w_{p(\text{specific})} = 0.1706 \, v^{1.875} \, L^{-0.125} \quad (14)$$

Fig. 10 gives two examples each for speeds of 5 m/s and 8 m/s. It also contains Gümbel's graphic presentation, which must lead to the same result, though by a more uncertain route.

It may be here remarked that Gümbel's statement, that we can assume the specific resistance to be practically constant at the corresponding distance from the leading edge, cannot be accepted as sufficiently reliable, in view of the new experiments. Table XIV is accordingly introduced here, because it continues, for still greater lengths than Fig. 10, the calculated results for the specific resistance at $v = 5$ m/s and 8 m/s. From this table it is evident that the drops between 10 and 20, between 20 and 100 and between 100 and 1000 are still quite large.

Table XIV.

a	b	c	d	e
Length in m	Resistance of surface strips, for 1 m surface width and various lengths, in kg. Speed, 5 m/s	Speed, 8 m/s	Specific surface resistance at a given distance from the leading edge, in kg/m^2 Speed, 5 m/s	Speed, 8 m/s
0.1	0.526	1.27	4.6	11.1
0.2	0.965	2.33	4.25	10.25
0.5	2.147	5.18	3.77	9.20
1	3.95	9.51	3.45	8.32
2	7.23	17.45	3.18	7.69
3	10.30	24.85	3.04	7.33
4	13.27	32.6	2.94	7.10
5	16.1	38.9	2.865	6.92
10	29.6	71.3	2.58	6.24
20	54.2	129.0	2.46	6.00
100	222	535	1.94	4.69
1000	1661	4005	1.46	3.51

This table also contains a numerical summary of the resistances of a one-meter-wide strip of various lengths, which gives some idea of the tremendous resistance, due to surface friction alone, which must be overcome by a large swiftly-moving ship.

From equation (13) for the resistance of surfaces of one meter width and various lengths at a speed of 1 m/s, the increase in resistance, due to the lengthening of a surface, can easily be determined. If the resistance of the foremost meter in length is called 100%, we can obtain an interesting view of the decrease in surface resistance with increasing length, by calculating, according to the following equation, the corresponding percentages for the last meter in length.

$$w' \text{ in \% of } \lambda = [L^{0.875} - (L - 1)^{0.875}]\ 100 \qquad (15)$$

We obtain, e.g., for

L =	1 m,	100%
L =	5 "	72%
L =	10 "	66%
L =	100 "	49%
L =	1000 "	37%

A surface length of 0.1 m gave 133% and a surface length of 0.01 m gave 178%, after pure turbulent friction had been attained. These numbers would hold good for any speed.

The evaluation of the experiments had been carried thus far, before the introduction of the resistance of the longitudinal edge. Since the evaluation has not only a chronological but also a comparative value, it has been here given unchanged.

10. Resistance of Longitudinal Edges.

The experiments described in Section 4 of the present article showed that the assumption of a resistance on the lower longitudinal edge of the towed plane was well founded. This adds to the difficulty of determining the pure surface resistance through the elimination of the displacement resistance only one more factor and, in any event, a numerical determination must be sought.

If we take from Fig. 9 the resistance of the 5-meter-long planes of various submersion depths at a speed of 1 m/s, hence under the assumption of the applicability of the discovered potential law of speed, and first calculate, after deducting the displacement resistance, the resistance of the planes for 1 cm submerged vertical perimeter, we then obtain an idea of the effect of the resistance of the longitudinal edge. This is carried out in Table XV and we recognize how in fact the specific resistance of the whole surface, corresponding to the assumed resistance of the longitudinal edge, would continually become smaller, the deeper the plane is submerged. Column e of Table XV gives the corrected resistance in Fig. 9 at 1 m/s for the various submersion depths. In order to enable us to determine therefrom the magnitude of the longitudinal-edge resistance and of the pure surface resistance, we still need the following assumption.

The experiments demonstrate the fact that, in any event, the longitudinal-edge resistance serves, together with the resistance of the planes, as the basis of the same potential law for the speed. Otherwise a different potential speed law would apply for the different widths of the planes. Hence,

$$w_k = \alpha\, v^{1.875} \tag{16}$$

The dependence of the longitudinal-edge resistance on the width and length of the plane is doubtful. Since the longitudinal-edge resistance is due to the same cause as the surface resistance, it may follow the same law in other respects and it appears simplest to regard it as the resistance of an enlarged surface, for the purpose of determining its numerical magnitude. Uncertain is the further temporarily necessary assumption that the longitudinal-edge resistance is the same for various submerged widths.

Table XV.

Resistance of a plane 5 m long and 1 cm thick, of various submerged perimeters, deducting from resistance, for pure turbulent condition, as read from the curves of the logarithm resistance diagram.

a	b	c	d	e
Depth submerged cm	Resistance at 1 m/s kg	Vertical submerged perimeter cm	Resistance per cm of perimeter kg	Corrected resistance at 1 m/s kg
2.5	0.055	6	0.00917	0.0558
5.0	0.097	11	0.00882	0.099
10.0	0.180	21	0.00857	0.178
15.0	0.260	31	0.00839	0.257
25.0	0.408	51	0.00800	0.408
50.0	0.800	101	0.00793	0.800

If w_a is the resistance of the 5 m plane at a speed of 1 m/s for the submersion width a; w_b, the resistance for a narrower submersion width b; u_a, the portion of the vertical perimeter wet in the first submersion case; u_b, the submerged portion of the perimeter in the second case; l, the length of the plane and x the increase in the width of the plane corresponding to the longitudinal-edge resistance with respect to the resistance calculation, we then have

$$w_a = \lambda l \, (u_a + x)$$

$$w_b = \lambda l \, (u_b + x)$$

$$\lambda = \frac{1}{l} \frac{w_a - w_b}{u_a - u_b} \tag{17}$$

$$x = \frac{w_a}{\lambda l} - u_a = \frac{w_b}{\lambda l} - u_b \tag{18}$$

Thus we obtain, as mean values,

$$\lambda = 0.1565 \text{ for } l = 5 \text{ m} \tag{19}$$

$$x = 0.0135 \text{ m} \tag{20}$$

i.e., the lower edge of the plane offers the same resistance as a strip 1.35 cm wide of a surface of infinite width but of the same length.

From the magnitude of x, we can perhaps also draw the conclusion that, for every degree of an edge angle, a perimeter increase of 0.000075 m must be introduced for the determination of the surface resistance. A wire of vanishingly small diameter, moved longitudinally through water, would accordingly meet the same resistance as a longitudinal strip 2.7 cm wide on an infinitely wide and very thin plane moving in a straight line with uniform speed in its own plane. (The longitudinal-edge resistance, in determining the resistance of keels, especially of stabilizing keels, would have to be found by experiments with models.)

It must be acknowledged that the mathematical determination of the undoubtedly existing longitudinal-edge resistance yet stands on a very uncertain basis, but the experiments offer no other solution. Unfortunately the resistance experiments with the brass plane failed, because the wide plane, probably due to the vibrations (which even resulted several times in the collapse of the whole plane), offered too great resistance.

11. Pure Surface Resistance.

After learning that, as a matter of fact, along with the displacement resistance, a longitudinal-edge resistance further increases the difficulty of determining the formulas and coefficients for the pure surface resistance, we will now briefly consider the improvements undertaken in the second direction. Although the evaluation of the experiments has already been undertaken without such consideration, this was done, as already stated, firstly, because this article is intended to give, to a certain extent, the chronological development of the whole matter and, secondly, to connect up with the earlier experiments.

Through the assumption of a longitudinal-edge resistance, even the greatly differing W. Froude values for λ are brought somewhat nearer the ones now found (Fig. 9), since it is known that W. Froude would have had to take the longitudinal-edge resistance twice into account, because his planes were towed entirely submerged. If we further consider that Froude disregarded both the resistance of the submerged supports (which were, however, well sharpened) of the planes both fore and aft, we can effect a further slight improvement in his results, but, since the longitudinal-edge resistance is only 2.8% of the total resistance and the other omissions affect the results still less, the total divergence of about 13% must be chiefly explained by other causes. The too great thinness of his planes can probably be regarded as the cause of the great distortions, since such distortions were observed in the Dresden experiments, with twice the thickness. At the same time, Froude, whose whole apparatus was not so stable as the modern, doubtless had to contend with much stronger vibrations and perhaps also with speed variations, especially as the towing was done by means of a rope actuated by a steam engine.

For our experimental results, the assumption of a longitudinal-edge resistance considerably improves the values of the smallest similar plane of 1.25 m length, since this plane was the narrower.

The calculation of $\zeta = \frac{W}{\rho v^2}$ has not yet been discussed, since the potential value of $\left(\frac{vl}{\nu}\right)$ was not 1.875 at first, but somewhat greater. This was chiefly due to the fact that the resistances, first obtained for the larger plane, were too large. The values of ζ are given in Table XVI and plotted in Fig. 11.

All the ζ values fall almost exactly on a curve, thereby demonstrating the applicability of the law of similitude to both cases under consideration. The temperature differences were unfortunately too small to give reliable numerical values of their effect.

Table XVI.

a	b	c		d
Speed in m/s	$\frac{\frac{v\,l}{\nu}}{10^7}$	$\frac{\zeta}{10^{10}}$	$\frac{\frac{W}{\rho v^2}}{10^{10}}$ (Pure surface resistance)	$\frac{\zeta}{10^{10}}$ (Surface resistance + edge resistance)
Plane 1.25 m long; temperature of water 10.2°C $\nu = 1.30 \times 10^{-6}$				
1	0.0962	0.0079		0.00871
2	0.1924	0.063		0.0697
3	0.2885	0.147		0.1626
4	0.3849	0.252		0.279
5	0.4820	0.389		0.430
6	0.5770	0.538		0.595
7	0.6735	0.716		0.792
8	0.7700	0.920		1.016
Plane 2.5 m long; temperature of water 9.9°C $\nu = 1.311 \times 10^{-6}$				
1	0.19075	0.0597		0.06275
2	0.3815	0.223		0.234
3	0.5721	0.472		0.496
4	0.763	0.83		0.872
5	0.954	1.24		1.307
6	1.145	1.72		1.810
7	1.335	2.37		2.480
8	1.527	3.19		3.35

Table XVI (Cont.)

9	b	c		d
Speed in m/s	$\frac{\frac{v\,l}{\nu}}{10^7}$	$\frac{\zeta}{10^{10}}$	$\frac{\frac{W}{\rho\,\nu^2}}{10^{10}}$	$\frac{\zeta}{10^{10}}$
		(Pure surface resistance)		(Surface resistance + edge resistance)
	Plane 5 m long; temperature of water 9.7°C $\nu = 1.32 \times 10^{-6}$			
1	0.3875	0.219		0.2253
2	0.7750	0.845		0.867
3	1.1620	1.795		1.842
4	1.550	3.09		3.170
5	1.9375	4.68		4.80
6	2.325	6.53		6.70
7	2.712	8.79		9.02
8	3.10	11.50		11.8
	Plane 7.5 m long; temperature of water 10.7°C $\nu = 1.29 \times 10^{-6}$			
1	0.5815	0.480		0.49
2	1.1625	1.861		1.89
3	1.744	3.931		4.0
4	2.325	6.75		6.87
5	2.907	10.30		10.48
6	3.489	14.45		14.70
7	4.070	19.45		19.8
8	4.650	25.20		25.6
	Plane 10 m long; temperature of water 7.1°C $\nu = 1.415 \times 10^{-6}$			
1	0.707	0.695		0.702
2	1.414	2.54		2.57
3	2.121	5.60		5.67
4	2.828	9.85		9.98
5	3.535	15		15.2
6	4.242	20.7		20.99
7	4.949	27.6		28.0
(7.5)	5.303	32.2		32.6

As shown by Fig. 9, where the ζ values for the tested similar planes are plotted in logarithmic distribution against the values, the ζ line has exactly the same ascending slope as the resistance lines with respect to the speed and length distribution. Hence the potential value $\left(\frac{v\,l}{\nu}\right)^{1.875}$ applies also for ζ and indeed for the tested form of plane, since, for $\left(\frac{vl}{\nu}\right) = 10^7$, we can read $\zeta_p = 1.40 \times 10^{10}$ and, for $\frac{vl}{\nu} = 1$, we can read

$$\zeta_p = \frac{1.4 \times 10^{10}}{\left(\frac{v\,l}{\nu}\right)^{1.875}} = \frac{1.4 \times 10^{10}}{(1 \times 10^7)^{1.875}}$$

$$\zeta_p = 0.00105 \left(\frac{v\,l}{\nu}\right)^{1.875} \tag{21}$$

This equation for the pure surface resistance would hold good for all planes whose length is 20 times their width and whose thickness is 1/25 of their width, i.e., if $l = 20$ b and $d = b/50$, or if $u = 1/9.804$.

Hence, if $u = l$ (i.e., if the submerged area = l^2),

$$\zeta = 9.804\ \zeta_p$$

$$\zeta = 0.01030 \left(\frac{v\,l}{\nu}\right)^{1.875} \tag{22}$$

The pure surface resistance then becomes

$$w_{l^2} = 0.01030 \left(\frac{v\,l}{\nu}\right)^{1.875} \frac{\gamma}{g}\ \nu^2 \tag{23}$$

$$w_{l^2} = 0.01030\ (v\,l)^{1.875}\ \frac{\gamma}{g}\ \nu^{0.125} \tag{23a}$$

If we should wish to introduce the surface F itself, we would have to divide the last equation by l^2 and we would obtain the general equation for pure surface resistance

$$w = 0.01030 \times l^{-0.125} \times v^{1.875} \times \frac{\gamma}{g}\ F\ \nu^{0.125} \tag{24}$$

This equation would hold good for all kinds of fluids and all temperatures, when smooth surfaces are moved. The development of this formula is very similar to the already improved Froude formula, except that it includes the effect of the specific gravity, of the acceleration due to gravity and of the temperature. It affords the possibility of determining the pure surface resistance through the elimination of the displacement and longitudinal-edge resistance. It yields somewhat greater values than the similarly derived Blasius formula which reads

$$\left(w = 0.0123\ l^{0.136}\ v^{1.864}\ \frac{\gamma}{g}\ F\ \nu^{0.136}\right)$$

This is due to the fact that Blasius used the writer's Dresden measurements, which, as already stated, gave somewhat smaller values. Moreover, only much smaller values could be considered and the temperature measurements were lacking.

The general equation found for the pure surface resistance enables the determination of the change in the same with rising temperature of the fluid. The resistance would vary in the ratio $\left[\frac{\nu}{(\nu)}\right]^{0.125}$ and it is thus found that a resistance change of about 0.36% would take place between 5 and 10°C and of about 0.31% between 10 and 20°C for every degree's change in the temperature of the water (according to Landolt-Börnstein-Roth's "Physikalische Chemische Tabellen," 1912. Compare Fig. 12*).

* Fig. 12 contains curves which were calculated from the η and γ values given in "Landolt-Bornstein-Roth phys. chem. Tabellen 1912." The η values are there given in the C.G.S. system, the forces, therefore, in dynes. Hosking's 1909 values (instead of Thorpe and Rodger's values, which were formerly much used) were employed in the calculation of ν for fresh water, because Hosking's values are the latest and agree well with Slotte's (1883). Krümmel and Rup-

(Cont. bottom next page)

Previous resistance and specific resistance formulas would have to be changed in their coefficients, in order to allow for the edge resistance. We now find

$$\lambda = 0.193 \quad \text{for} \quad 10^{\circ}C \tag{25}$$

and therefrom the pure surface resistance for water at $10^{\circ}C$ (See equation (11)).

$$w_{10} = 0.193 \times l^{-0.125} \quad F \times v^{1.875} \tag{26}$$

The specific resistance at the distance L from the leading edge would then be (See equation (14a))

$$w_{sp} = 0.193 \times 0.875 \; v^{1.875} \times L^{-0.125} \tag{27}$$

$$w_{sp} = \frac{0.1689}{L^{0.125}} \; v^{1.875} \tag{28}$$

If we wish to find the specific resistance for any fluid and temperature, it is only necessary to vary the composition of the formula according to equation (24) and we obtain the generally applicable equation

$$w_{sp} = 0.01030 \, \frac{\gamma}{g} \, \nu^{0.125} \times 0.875 \; v^{1.875} \; L^{-0.125} \tag{29}$$

$$= \frac{0.0090125}{L^{0.125}} \, \frac{\gamma}{g} \, \nu^{0.125} \times v^{1.875} \tag{30}$$

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*(Cont. from p. 57) pin's values (1905) were employed for sea water. The η values vary greatly with the different sources. Consequently, the ν values for air, in Fig. 12, can be regarded as utilizable only to one decimal place of the number $\times 10^{-5}$ for m^2/sec., or to two decimal places in the values for cm^2/sec. The intermediate values of ν (between fresh water and sea water of γ =1026=3.5% salt content) for sea water of other specific gravities are proportional to the increase in content of salt or in specific gravity.

12. Comparison with the Values Obtained from R. E. Froude's Formula.

In the towing laboratories for ship models, the computations are made almost exclusively with R. E. Froude's coefficients worked out, on the basis of his own and his father's experiments, according to his formula for the surface resistance

$$w = \gamma \lambda F v^{1.825}$$

R. E. Froude gives his coefficient, which he designates with O_m for the model and O_s for the ship, in another form and in the English system of measuring units (Transactions of the Institution of Naval Architects, 1888: "On the constant system of notation of results" etc.). If, from this, we compute λ_m and λ_s in the metric system and plot the individual results logarithmically against the lengths l (Fig. 13), we see that the Froude values for λ represent two different functions of l. From 1-10 m, the equation for l would read

$$\lambda = \lambda_1 \, l^{-0.130} = 0.213 \, l^{-0.130} \tag{31}$$

and above 30 m

$$\lambda = \lambda_1 \, l^{-0.027} = 0.161 \, l^{-0.027} \tag{32}$$

Neither formula agrees with the law of similitude, since the expression $f\left(\frac{v l}{\nu}\right)$ would not be nondimensional.

From the fact that R. E. Froude based his coefficients for a ship on another function of l, at least above 30 m, he evidently wished to make allowance for the assumed rough condition of the surface. The noteworthy point in this connection is that then, for short lengths, the coefficient would be smaller for a ship than for a model. In order to avoid this, Froude seems simply to have converted the λ curve for the ship into the curve for the model.

Perhaps this is the best place to say something further concerning Froude's values.

In all countries of the world, they are regarded as the basis for computing the friction HP. in model experiments. (Only in America, computations are made for ships over 33 meters long with Thidemann's values.) Since Germans gradually abandoned Froude's theory of constants and finally reverted to the old simple formula for frictional resistance $\gamma \lambda F v^{1.825}$, Froude's values, which were given only in the complicated form O_m and $O_s = \dfrac{1000\ \lambda\ L^{-0.0875}}{\left(\frac{4\pi}{g}\right)^{0.9125}}$ and the English system of measures, also went through the corresponding retrogression.

They were converted by Schütte into the meter-kilogram-second system (Zeitschrift für Schiffbau, Vol. II, 1900-1901, p. 208). Bruckhoff then published a simplification of the computation formula for the friction HP. with these values (Zeitschrift "Schiffbau," Vol. VI, 1904-1905, p. 67). At the same time, the writer had gone still further and had entirely abandoned Froude's theory of constants and evolved a formula for the friction HP. in a new direct manner and also calculated, from the Froude O_m and O_s values, the λ values for the meter-kilogram-second system (See Table A).

Table A.

Friction values λ in kg per m² surface, dependent on the length l according to R. E. Froude.

l	λ	l	λ	l	λ
		1. For paraffin models of ships.			
0.50	0.2280	2.75	0.1879	5.00	0.1727
0.75	0.2198	3.00	0.1857	5.25	0.1716
1.00	0.2132	3.25	0.1836	5.50	0.1706
1.25	0.2079	3.50	0.1817	5.75	0.1696
1.50	0.2034	3.75	0.1799	6.00	0.1687
1.75	0.1994	4.00	0.1782	6.25	0.1679
2.00	0.1960	4.25	0.1767	6.50	0.1672
2.25	0.1930	4.50	0.1752	6.75	0.1664
2.50	0.1903	4.75	0.1739	7.00	0.1658
				7.25	0.1651
				7.50	0.1645
		2. For ships with well painted surfaces.			
10	0.1590	55	0.1442	100	0.1422
15	0.1537	60	0.1439	110	0.1418
20	0.1508	65	0.1436	120	0.1415
25	0.1488	70	0.1434	130	0.1412
30	0.1474	75	0.1432	140	0.1408
35	0.1464	80	0.1430	150	0.1405
40	0.1457	85	0.1428	160	0.1402
45	0.1450	90	0.1426	170	0.1399
50	0.1446	95	0.1424	180	0.1396
		From here down the values were obtained by extending the adjusted curve.			
190	0.1394	230	0.1383	270	0.1374
200	0.1391	240	0.1380	280	0.1372
210	0.1388	250	0.1378	290	0.1369
220	0.1386	260	0.1376	300	0.1367

The above values were published by Schaffran along with the simplified formula of the writer ("Schiffbau" Company: Schaffran, "Die Versuchsmethoden der Koniglichen Versuchsanstalt für Wasserbau und Schiffbau." See also "Zeitschrift Schiffbau," Vol. XVI, 1914-1915, pp. 223 and 382.

After the values had thus found their way to publicity without the aid of the writer, it is perhaps opportune at this point to explain that these values were taken from R. E. Froude only up to lengths of about 183 m (600 feet). Only thus far could the values be converted, since Froude did not give values for greater lengths. With the considerable increase in the length of ships, we needed to know the value of the resistance for greater lengths. Froude's values could not be employed in an equation. At that time the writer did not know that they followed two equations and haste was necessary. The curve of the λ values, plotted against the lengths as abscissas, was simply extended according to judgment. Thus the values for lengths above 180 m, given by Schaffran and here repeated, were obtained by extrapolation. The deviation is not great, however, as is shown by the following Table B.

Table B

Length in mm	λ according to the curve	λ according to equation
30	0.1474	0.1484
50	0.1446	0.14486
100	0.1422	0.1422
150	0.1405	0.1403
180	0.1396	0.13994
200	0.1391	0.13954
250	0.1378	0.1387
300	0.1367	0.13704

The comparison of a few computation results, according to the formula of the younger Froude and according to the new formula, may now be of interest. Table XVII gives a brief comparison for various speeds and lengths and shows that Froude's values for short lengths, up to 10 m, are approximately equal, but deviate rapidly upward for greater lengths. For a length of 300 m the deviations of 20% are so great as to render doubtful the utility of either formula.

Table XVII.

Calculated resistance (in kg) of a surface strip of 1 m width and different lengths at different speeds.

a Length m	b λ	Resistance in kg at speeds of					
		c 1 m/s	d 2 m/s	e 5 m/s	f 10 m/s	g 15 m/s	h 20 m/s
According to R. E. Froude, without mention of water temperature.							
1	0.2132	0.2132	0.756	4.025	14.26	29.83	49.25
10	0.159	1.59	5.65	30	106.2	223	375
100	0.1422	14.22	50.5	268	951	1992	3360
300	0.1367	41.01	145.6	775	2740	5750	9680
According to the new formula at a water temperature of 10°C.							
1	0.193	0.193	0.725	3.95	14.47	30.9	53.1
10	0.1448	1.448	5.425	29.6	108.5	232	398
100	0.1085	10.85	40.7	222.2	814	1740	2985
300	0.0947	28.4	106.5	580	2130	4550	7800
From Dresden experiments by Blasius without mention of water temp.							
1	0.200	0.200	0.730	4.02	14.6	31.1	53.4
10	0.1462	1.462	5.20	29.4	107	226	390
100	0.1069	10.69	39.0	207.2	781	1660	2850
300	0.0923	27.69	101.0	556	2012	4300	7390

The advantages of the new values are as follows:

1. The considerably greater speed range of the new experiments;

2. The more frequent repetitions of the experiments at dif-

ferent times;

3. The improved experimental apparatus;

4. The fact that the values are smaller. Faults in the adjustment of the planes or in the smoothness of the surface would always cause upward deviations;

5. The new values agree more closely with the law of similitude;

6. Froude's neglect of the displacement and edge resistance;

7. The good agreement with the results of the writer's experiments in Dresden-Uebigau, where the speed range was also somewhat greater than Froude's.

All of the above are reasons for abandoning Froude's values. The following reasons are to the contrary:

1. Nearly all the available results of towing experiments with models were calculated with them;

2. New experiments with surfaces, which were covered with antifouling paint and hence furnished the transition from the smooth surface of the models to the probably rougher surface of the ship,* need to be completed;

3. Proof is lacking, as to how the surface resistance varies for curved surfaces and for other than rectangular surfaces and especially the resistance for objects (See Section 13, "McEntee's Experiments").

*In towing experiments, this does not enter into the question for small scales, since the smoothness of the model and of the ship's surface must be similar.

4. Experiments are lacking with large ships, executed in a manner similar to the ones performed by the older Froude with the "Greyhound."

13. William McEntee's Experiments.

The results of resistance experiments by William McEntee with planes in two lengths were published in the "Transactions of the Society of Naval Architects and Marine Engineers" in 1915. The shorter steel planes 3.05 m (10 ft.) long and 0.61 m (2 ft.) wide and weighing 4.536 kg (from which the thickness, not given by McEntee, was computed to be about 3 mm) were coated with anti-fouling paint and were towed both in the freshly painted condition and also after various periods of exposure to the action of sea water. The fouled paint was then carefully removed, the plane freshly painted and again towed in the painted condition. For the freshly painted (of course well dried) plane, the speed exponent was found to be 1.88 and the coefficient $\lambda = 0.17$ at speeds up to 4.6 m/s, values which agree remarkably well with the value here given for lacquered surfaces. We may therefore assume with probably sufficient accuracy, that the values found for lacquered surfaces also hold good for smooth ship hulls.

Experiments previously performed by the writer demonstrated that even smooth paraffin and smooth plaster made from pure cement are to be regarded as having the same value as lacquer with respect to the surface resistance.

Furthermore, Mr. McEntee's experiments demonstrated that, with these shorter planes of 3.05 m length, the surface resistance increased three to four-fold, when the planes were exposed from five to twelve months to the action of sea water. Cleaning and fresh paint, however, restored the resistance to almost exactly what it was at first. These experiments do not demonstrate, however, that this great increase in resistance, caused by the corrosion of the paint and by the barnacles, would be in the same ratio for longer surfaces. It is, instead, probable that with increase in length, there would be a considerable reduction in the percental resistance increase,* because such a strong water current would be generated by the front portion of the surface that less would be left for the rear portion to do, than in the case of a smooth surface. Hence the need of experiments with such rough planes of the greatest possible difference in length.

The longer planes, apparently wooden, which McEntee tested, were 6.1 m (20 ft.) long, 0.61 m (2 ft.) wide and 19 mm thick, and were lacquered and towed at speeds up to 5.66 m/s (11 knots per hour). The exponents 1.883 and 1.886 and the coefficient λ converted to 0.1435 for m, kg, and sec., likewise agreed well with those obtained in the new experiments.

McEntee's long planes were not rectangles, but oblique-

* The experiments of William Froude, with planes roughened by sand of varying coarseness, were not satisfactory, since, as McEntee demonstrated, no such differences were obtained as with the smooth planes. In this case also, as in general, Froude's experiments did not satisfy modern speed requirements.

angled parallelograms, having an angle of 30°, with the lower edge projecting forward. Moreover, they were towed entirely submerged, with the upper edge, however, only about 20.3 mm under water.

The attempt to reduce the surface resistance, with graphite, oil or soap, below that for lacquer, was also demonstrated by McEntee to be in vain.

14. Contemplated Continuation of the Experiments.

The results of these experiments seem to indicate the desirability of continuing them in a different direction. Preparations have been made for experiments with still longer planes and for investigating the motion of the water around the moving plane, both laterally and also longitudinally at various distances from the leading edge, underneath and at various heights. Hence no report is here given on the motion of the water and on the conclusions which might be drawn from the experiments already made. Further special experiments are contemplated on the longitudinal-edge resistance.

Experiments with surfaces of other kinds and shapes and especially with the surface resistance of bodies are important for ship designers. Since a thin plane, as well as a thicker body, in its motion through the water, generates, at a certain distance, a lateral flow toward the rear, this investigation will be very comprehensive and not very simple. The measurement of the water's velocity will be rendered more difficult by the formation of waves and, since nearly all investigators have hitherto disregarded this factor, nothing of use is given in the literature on the subject.

There also remains to be investigated the effect on the surface resistance produced by reducing the cross-section of the water.

Summary

The proof of the validity of the Reynolds law of similitude for the surface resistance of planes has been developed with an accuracy hitherto unattained and for a large range of lengths and speeds. It has been shown that, in addition to the form resistance, the resistance of the longitudinal edges must be taken into account.

A comparison with Froude's values showed that the new values are considerably smaller. They agree well, however, with the results of the writer's Dresden experiments and with the values obtained by Mr. McEntee.

Further experiments are desirable and are contemplated.

16. Appendix.

Brief deduction of the Reynolds law of similitude for plane surfaces towed longitudinally in a straight line through water.*

It is known that the law of similitude can often yield numerical results in important dynamic problems, for which the ordinary method of mathematical deduction is not available. It reproduces motion phenomena by means of a model, determines the magnitude of the forces required to produce the motion and predicts the force required for the production of similar phenomena on a larger scale. The greatest use of it will probably be made in ship designing. The simplest cases are those of surfaces moving laterally or longitudinally through water. We will here discuss only the latter case.

The task might perhaps be given the law of similitude to determine the magnitude of the force required to move a large, rectangular, smooth, lacquered, thin plane of given dimensions, at a given uniform speed, completely submerged and parallel to the surface of the water.

As the experimental model, there had to be constructed an exactly similar plane λ** times as small in all its dimensions and even in its degree of roughness. In towing this plane, all the paths of the corresponding water particles have to be exactly

* See also "Sammelheft 1 des Ausschusses für technische Mechanik des Berliner Bezirkvereins deutscher Ingenieure," Berlin, 1919: M. Weber, "Die Grundlagen der Aehnlichkeitsmechanik und ihr Ausbau zu einem periodischen System der Modellzesetze."
** The symbols are taken from Weber.

similar to those produced on the large scale and have likewise to be reduced in the ratio λ. It is also desirable that the duration period of the comparable phenomena for all particles should have the same fixed scale τ. It is also desirable that the water cross-sections, unless so large that they can be regarded as practically infinite, should have the same ratio λ to one another. Corresponding distances and durations should therefore be observed.

If the length ratio λ is arbitrarily chosen, it then follows, from the requirement of similarity for the paths, that the time ratio τ must correspond and vice versa.

Mechanical similarity requires, in contradistinction to kinetic similarity (which prescribes the compulsory paths for the mass elements), that all mass elements carry out their motions freely under the action of natural forces.

In the present case, therefore, the acceleration forces of the magnitude ("mass times acceleration") and the forces of viscosity are alone operative. It even follows that the forces of viscosity must equal the forces of acceleration, since, when we assume, in advance, such a vanishingly small thickness of the plane that the form resistance vanishes, then the water particles can be accelerated only by the effect of the viscosity, i.e., the whole force of viscosity must be employed for mass acceleration. Corresponding to their manner of working, the viscosity forces are also termed internal-friction forces.

The following symbols can now be introduced:

a Symbol for full-sized object and for model			b Technical unit ~~of mass~~
Length	L	l	m
Time	T	t	sec.
Force	K	k	kg
Surface	F	f	m^2
Volume	Vol	vol	m^3
Speed	V	v	$\frac{m}{sec.}$
Acceleration	B	b	
Mass	M	m	
Density = $\frac{\text{unit of weight}}{\text{acceleration due to gravity}}$	(ρ)	ρ	
Weight unit	(γ)	γ	
Technical viscosity value	(η)	η	
Viscosity factor	(ν)	ν	

a Symbol for full-sized object and for model			c Conversion scale	d Similitude symbol
Length	L	l	λ	$L = l \lambda$
Time	T	t	τ	$T = t \tau$
Force	K	k	κ	$K = k \kappa$
Surface	F	f	λ^2	$F = f \lambda^2$
Volume	Vol	vol	λ^3	$Vol = vol \lambda^3$
Speed	V	v	$\frac{\lambda}{\tau}$	$V = v \frac{\lambda}{\tau}$
Acceleration	B	b		
Mass	M	m		
Density = $\frac{\text{unit of weight}}{\text{acceleration due to gravity}}$	(ρ)	ρ		
Weight unit	(γ)	γ		
Technical viscosity value	(η)	η		
Viscosity factor	(ν)	ν		

We can now write the magnitude of the molecular tension on the corresponding surfaces F and f as $(\eta)\frac{\delta V}{\delta N}$ and $\eta\frac{\delta v}{\delta n}$, when $\frac{\delta V}{\delta N}$ and $\frac{\delta v}{\delta n}$ denote the speed changes on the normal surface N and n. The internal friction forces acting on the corresponding surfaces F and f are therefore

$$K = (\eta)\frac{\delta V}{\delta N} F \text{ and } k = \eta \frac{\delta v}{\delta n} f.$$

Hence

$$K = \frac{K}{k} = \frac{(\eta)\frac{\delta V}{\delta N} F}{\eta \frac{\delta v}{\delta n} f} = \frac{(\eta)}{\eta} \frac{\lambda}{\tau\lambda} \lambda^2 = \frac{(\eta)}{\eta} \frac{\lambda^2}{\tau} \tag{1}$$

The ratio of the forces due to gravity is, however:

$$K = \frac{K}{k} = \frac{M\ B}{m\ b} = \frac{(\rho)}{\rho} \lambda^3 \frac{\lambda}{\tau^2} = \frac{(\rho)}{\rho} \frac{\lambda^4}{\tau^2} \tag{2}$$

These two expressions are equivalent and hence

$$\frac{(\eta)}{\eta} \frac{\lambda^2}{\tau} = \frac{(\rho)}{\rho} \frac{\lambda^4}{\tau^2} \text{ or } \tau = \lambda^2 \frac{(\rho)\,\eta}{\rho\,(\eta)}$$

or, if we introduce $(\upsilon) = \frac{(\eta)}{(\rho)}$ and $\upsilon = \frac{\eta}{\rho}$,

$$\tau = \lambda^2 \frac{\upsilon}{(\upsilon)} \tag{3}$$

Hence

$$T : t = \frac{L^2}{(\upsilon)} : \frac{l^2}{\upsilon} \tag{4}$$

The last equation is the Reynolds model law for corresponding periods of time. In considering exactly equivalent fluids at the same temperature, this equation is simplified to

$$T : f\, L^2 : l^2 \qquad (5)$$

If we wish to determine the speed ratio, we only need to write equation (3) in the following form:

$$\frac{\lambda}{\tau}\lambda \frac{\nu}{(\nu)} = 1$$

or

$$\frac{V}{v}\,\frac{L}{l}\,\frac{\nu}{(\nu)} = 1$$

or

$$V : v = \frac{(\nu)\, l}{L\ \nu} = \frac{(\nu)}{L} : \frac{\nu}{l} \qquad (6)$$

The last equation can be written in the following important form:

$$\frac{V\ L}{(\nu)} = \frac{v\, l}{\nu} = \psi \qquad (7)$$

ψ is nondimensional, since both enumerator and denominator have the same unit of measure m^2/sec. It is the nondimensional form of the Reynolds model law and is therefore independent of the units of measure employed.

According to equation (2), the ratio of the forces is

$$\kappa = \frac{K}{k}\,\frac{(\rho)}{\rho}\,\frac{\lambda^4}{\tau^2}$$

or

$$\kappa = \frac{(\rho)}{\rho}\,\frac{(\nu)^2}{\nu^2} = \frac{\dfrac{(\eta)^2}{(\rho)}}{\dfrac{\eta^2}{\rho}} \qquad (8)$$

The trend of modern experimental science is to express experimental results as far as possible in nondimensional forms.

Equation (7) contains this law for the speeds in terms of the divided scales and the viscosities. There still remains to be found the nondimensional expression

$$\zeta = f\ (\psi)$$

for the forces.

If we write equation (2) in the form

$$\kappa = \frac{K}{k} = \frac{MB}{mb} = \frac{(\rho)}{\rho}\ \lambda^2\ \frac{\lambda^2}{\tau^2} = \frac{(\rho)FV^2}{\rho\ fv^2} \tag{9}$$

we obtain the two inseparable equations

$$\left\{\begin{matrix} K = \alpha\ (\rho)\ F\ V^2 \\ k = \alpha\ \rho\ f\ v^2 \end{matrix}\right\} \tag{10}$$

in which α is a common coefficient. Equation (10) can also be written

$$\left\{\begin{matrix} K = \epsilon\ (\rho)\ L^2 V^2 \\ k = \epsilon\ \rho\ l^2\ v^2 \end{matrix}\right\}$$

Since $VL/(\nu) = vl/\nu = \psi$, we have

$$\left\{\begin{matrix} K = \epsilon\ (\rho)\ \psi^2 (\nu)^2 \\ k = \epsilon\ \rho\ \psi^2\ \nu^2 \end{matrix}\right\} \tag{11}$$

or, if we substitute ζ for $\epsilon\ \psi^2$,

$$\left\{\begin{matrix} K = \zeta\ (\rho)\ (\nu)^2 \\ k = \zeta\ \rho\ \nu^2 \end{matrix}\right\} \tag{12}$$

From the forces measured on the model, however, we obtain

$$\zeta = \frac{k}{\rho\ v^2} \tag{13}$$

which is also nondimensional

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

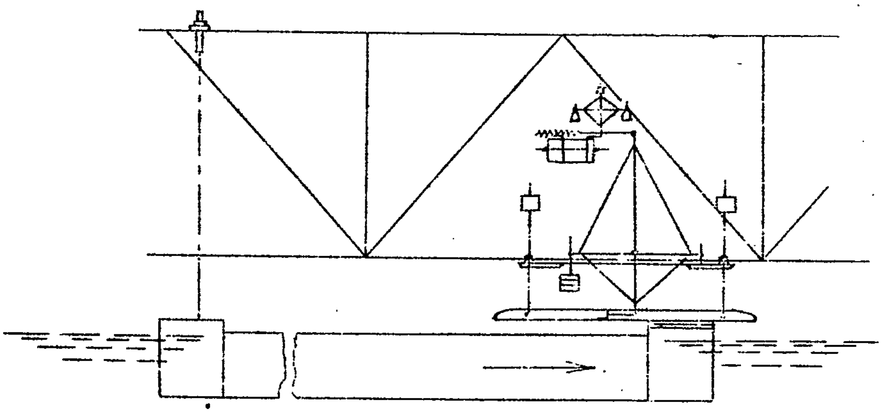

Fig.1 Diagram of device for towing small planes.

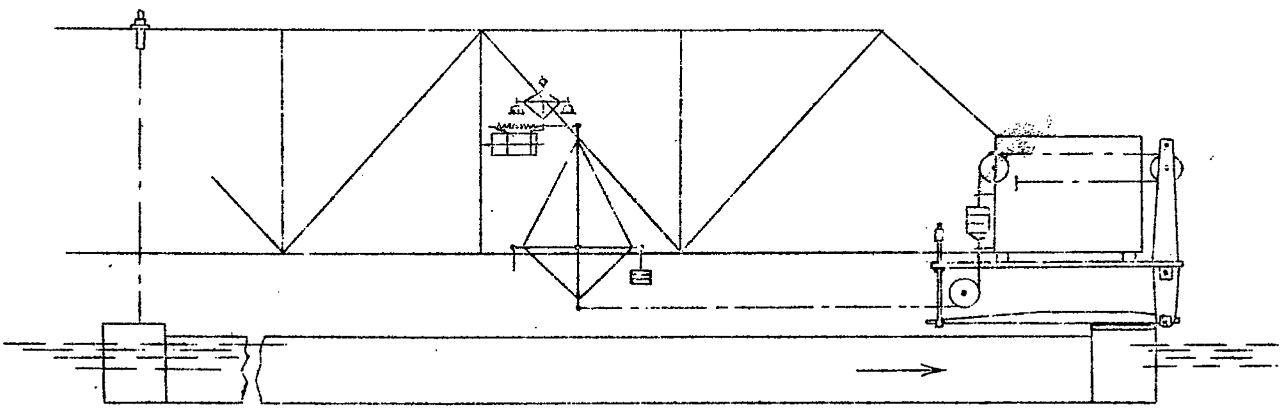

Fig.2 Diagram of device for towing large planes.

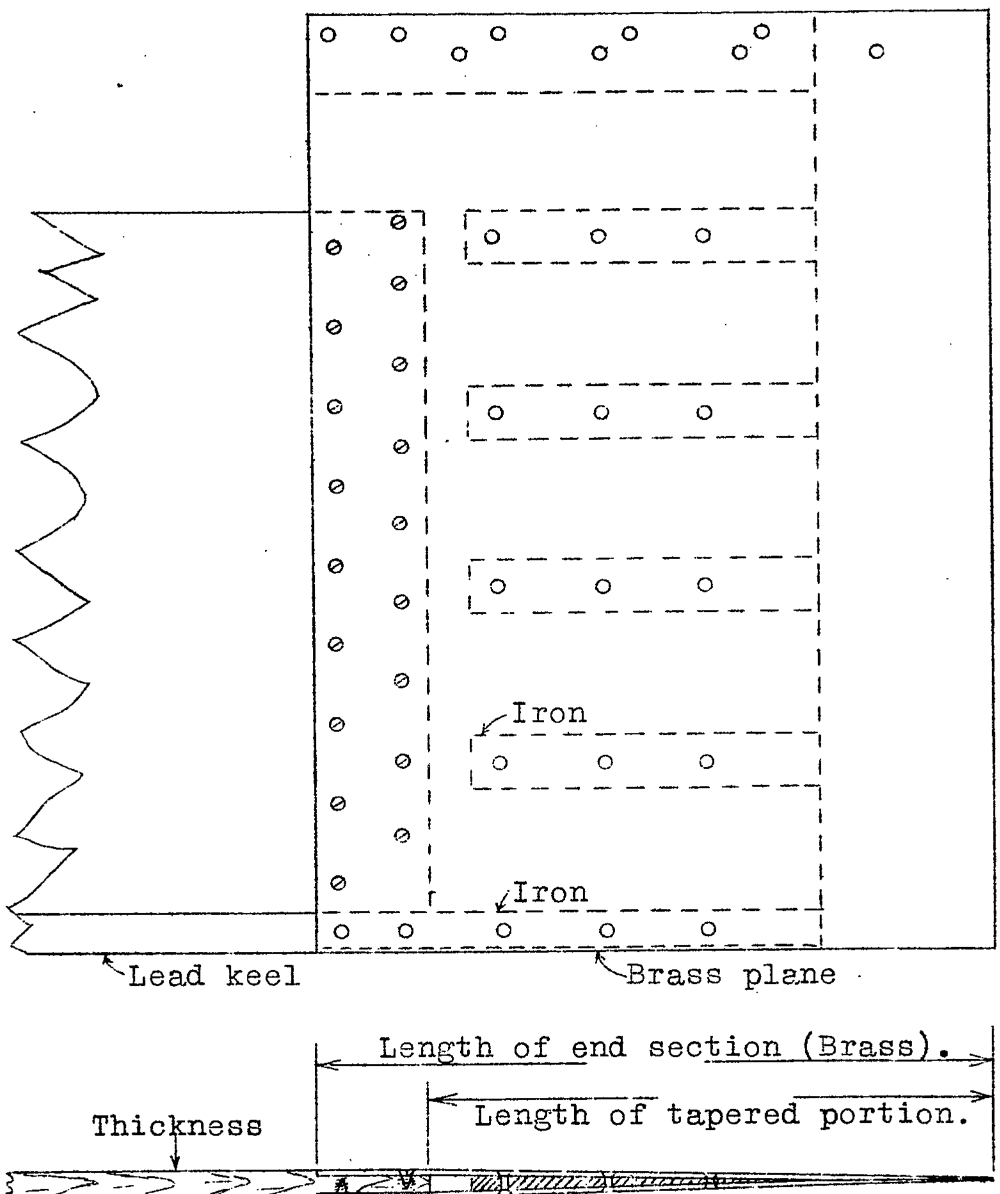

Fig.3 Brass end sections for the planes.

Curve		Thick, mm	Long, m	Submerged, mm	Lacquered	Towed	Date, 1916	Temp. of water °C	Symbol
10	Brass plate	3	1	200	"	"	4/28	10.7	
1	"	3	1	375	"	"	4/11	10.5	
4	"	3	1	375	"	"	4/14	10.08	
7	"	3	1	375	"	"	4/18	9.67	
8	"	3	1	375	"	"	4/20	9.67	
9	"	3	1	375	"	"	4/28	10.7	
2	End section	10	1	375	"	"	4/11	10.5	
3	"	12	1	375	"	"	4/13	10.48	
6	"	15	1	375	"	"	4/17	9.67	
5	"	20	1	375	"	"	4/15	10.22	

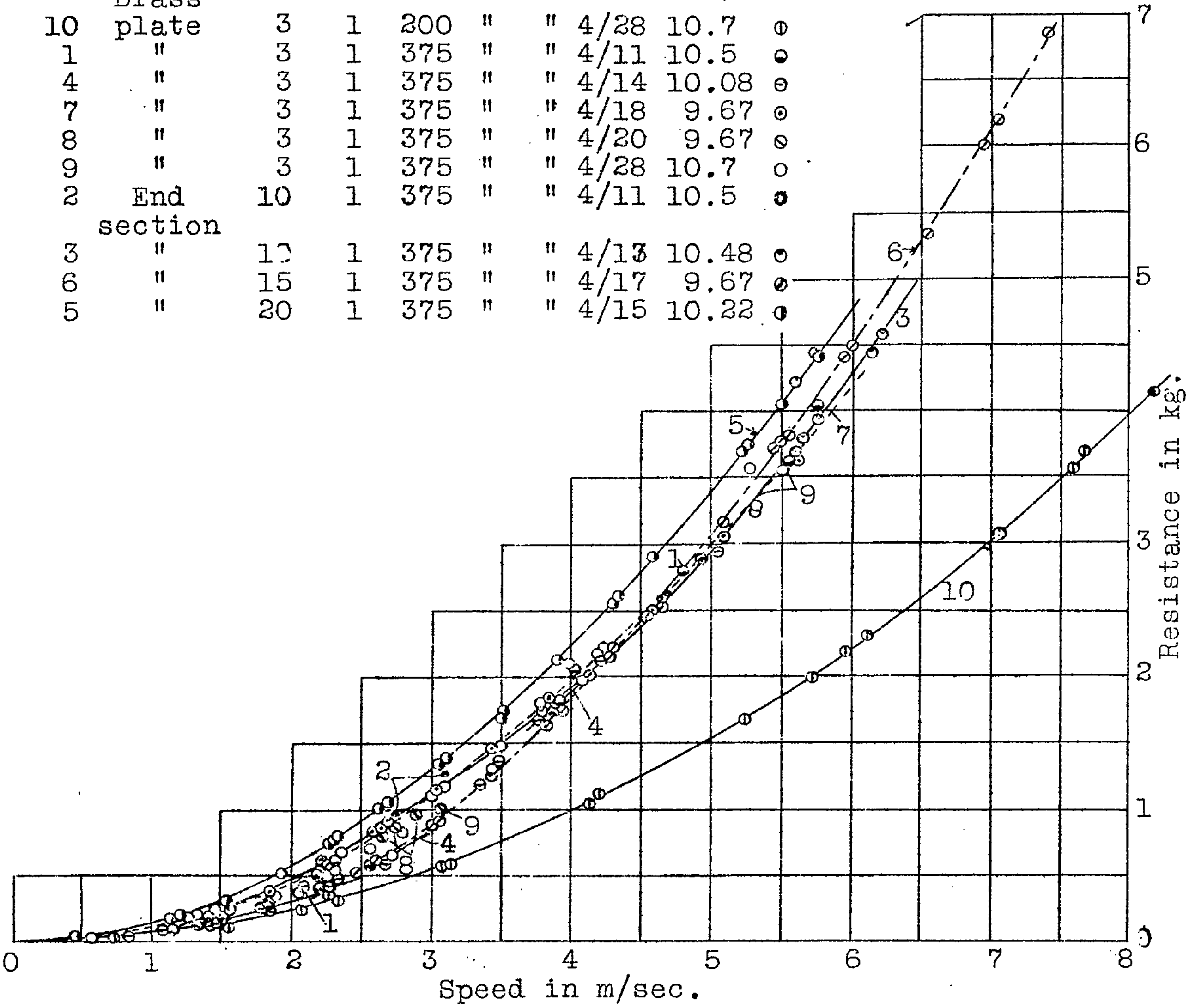

Fig.4 Resistance of brass planes and of brass end sections in kg.

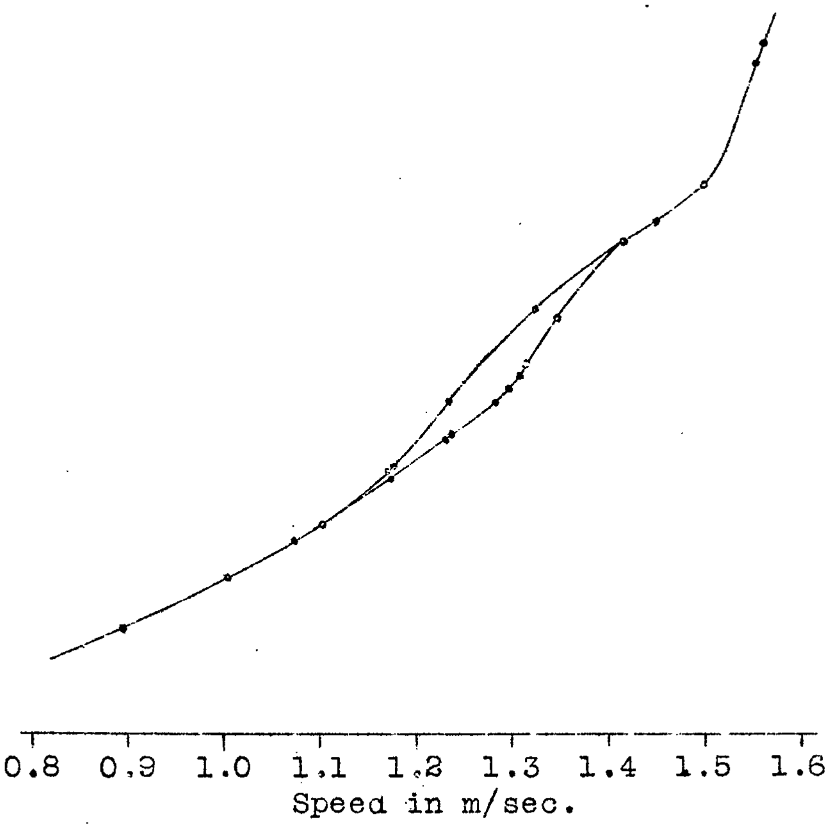

Fig.4a Deviation of resistance of a ship model 4.791 m long in deep water.

Curve	Plane length, m	Submerged, mm	Lacquered	Towed	Date, 1916	Temp. of water °C	Symbol
1	1.25	75	"	"	4/22	10.20	∘
2	2.50	125	"	"	4/21	9.90	∘
3	5.00	250	"	"	4/19	9.63	∘
3	5.00	250	"	"	4/20	9.67	•
4	5.00	500	"	"	4/25	10.20	+
4	5.00	500	"	"	4/26	10.31	•
4	5.00	500	"	"	5/1	10.71	∘
5	7.50	375	"	"	4/27	10.77	×
5	7.50	375	"	"	4/28	10.65	∘

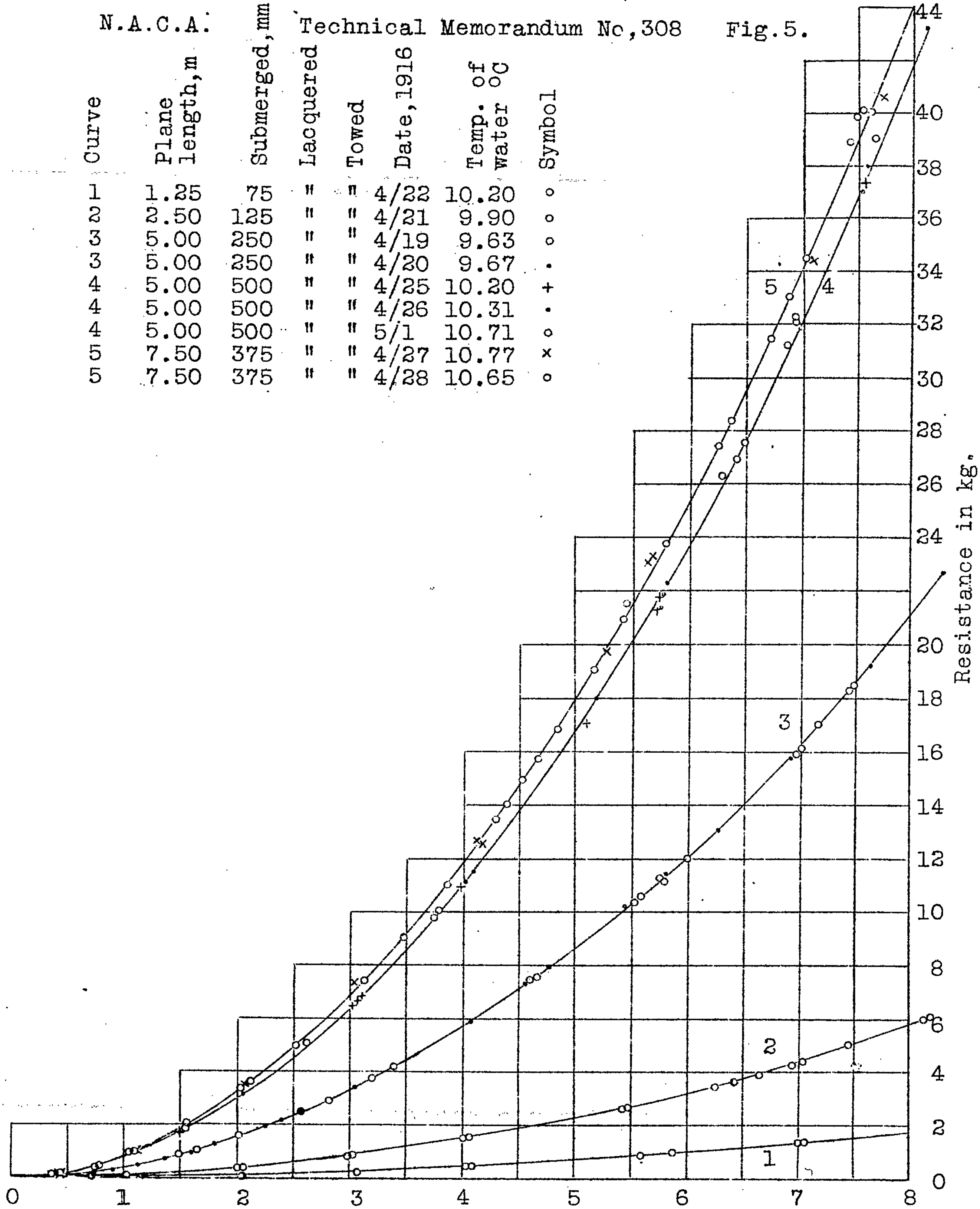

Fig.5 Resistance of planes in kg.

Curve	Plane length, m	Submerged, mm	Lacquered	Towed	Date, 1917	Temp. of water °C	Symbol	Without lead keel
1	5.00	25	"	"	4/7	7.70	∘	
1	"	25	"	"	4/18	8.49	•	"
2	"	50	"	"	4/6	8.00	∘	
2	"	50	"	"	4/19	8.40	•	"
3	"	100	"	"	4/6	8.00	•	
3	"	100	"	"	4/11	7.80	×	
3	"	100	"	"	4/19	8.40	∘	"
4	"	150	"	"	4/6	8.00	•	
4	"	150	"	"	4/20	8.40	×	"
4	"	150	"	"	4/21	8.45	∘	"
5	"	250	"	"	4/5	7.60	•	
5	"	250	"	"	4/7	7.70	×	
5	"	250	"	"	4/21	8.45	∘	"

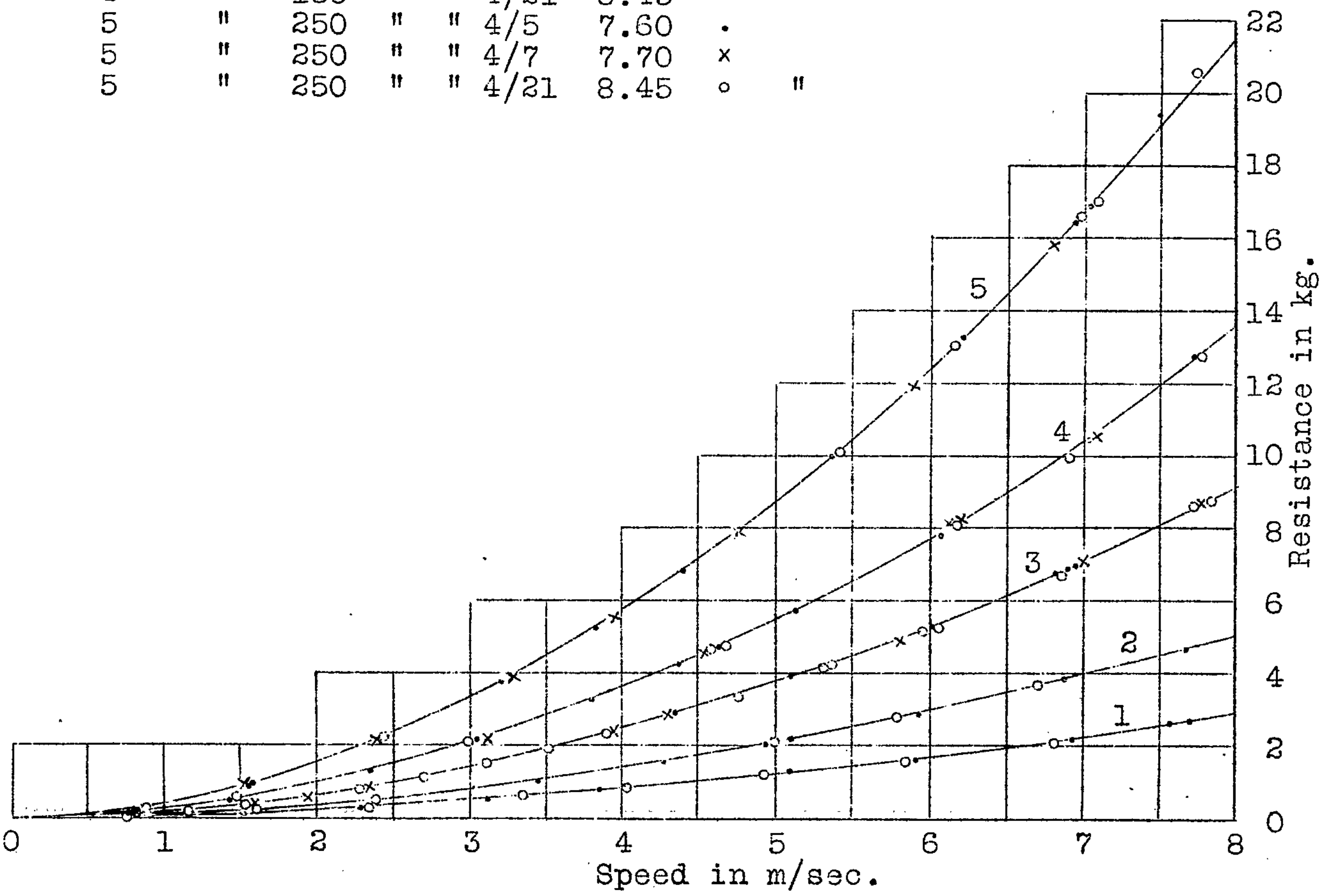

Fig.6 Resistance of planes in kg.

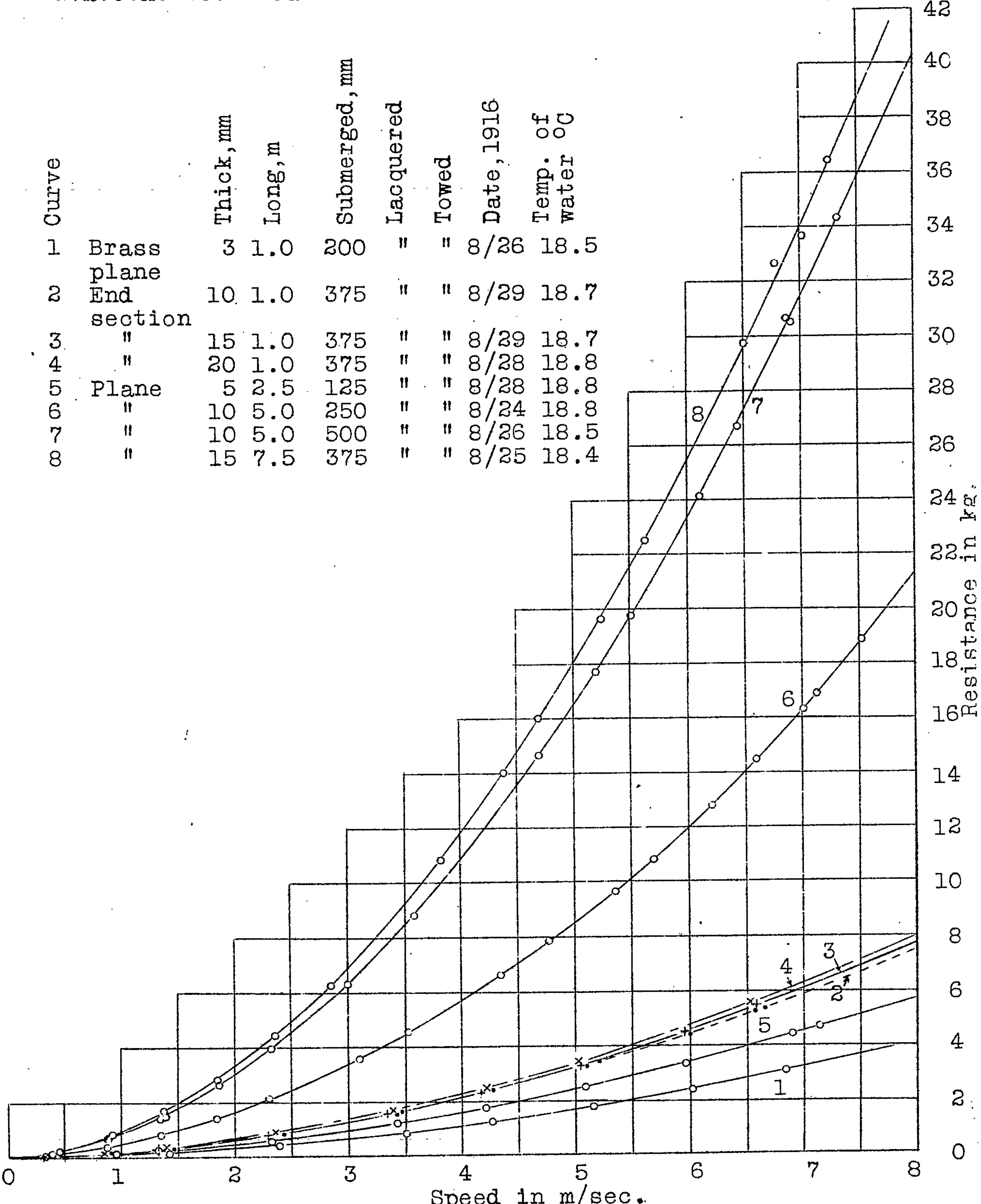

Fig.7 Resistance of planes and end sections in kg.

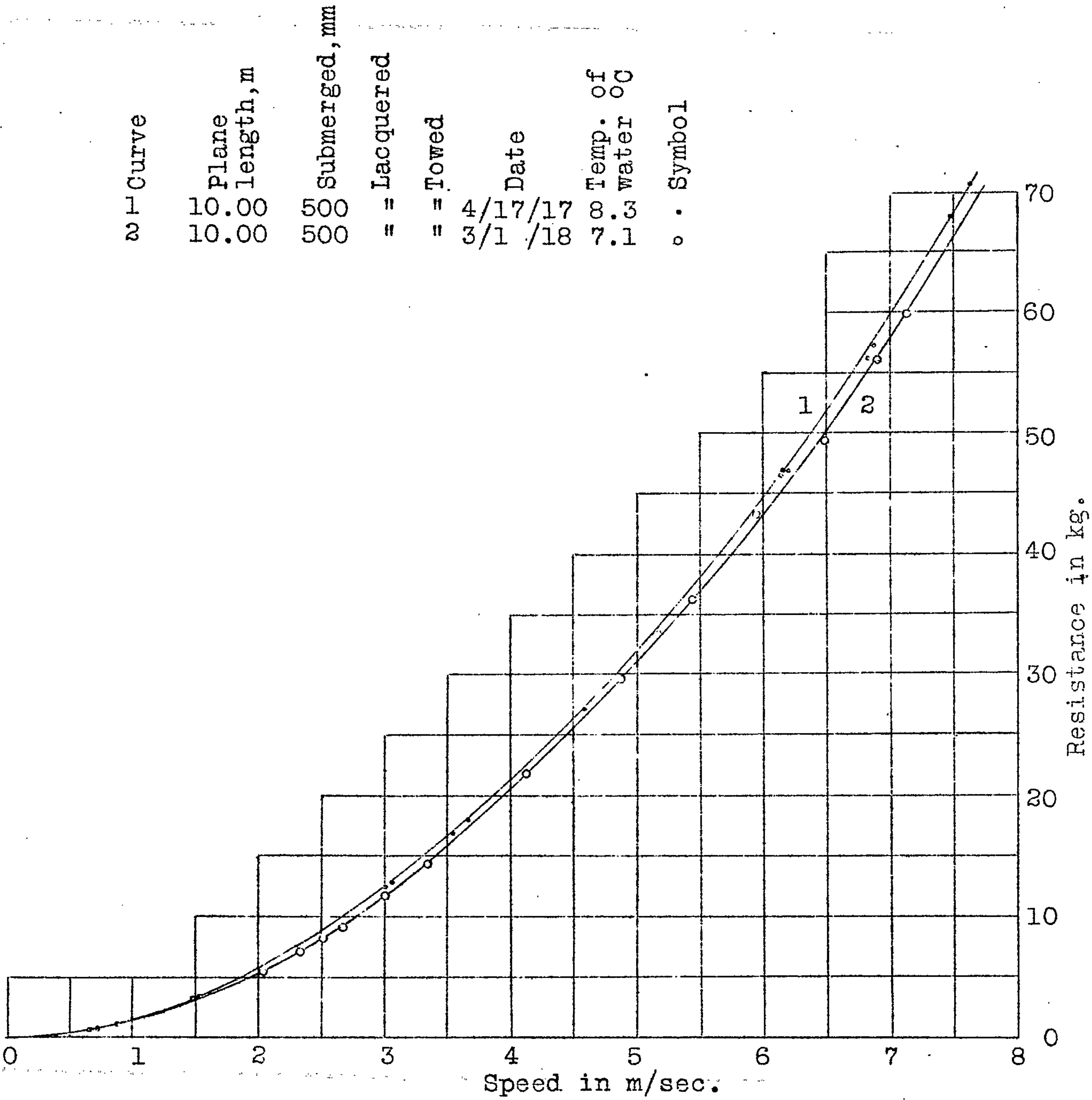

Fig.8 Resistance of planes in kg.

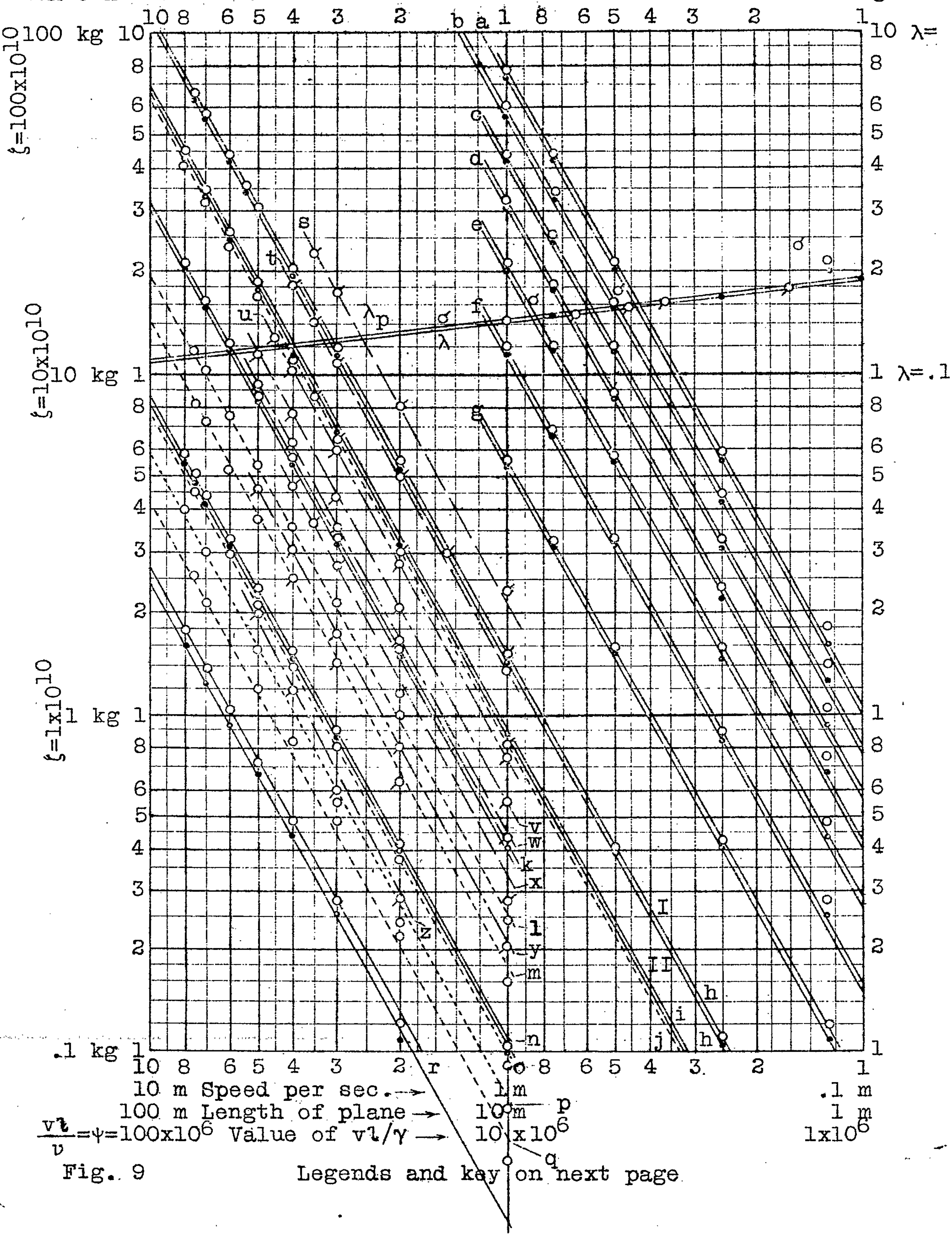

Fig. 9

Legends and key on next page

Resistance of planes — with edge resistance — Froude's, Dresden — λ ζ

Temperature of water 10°C

Without edge resistance
Similar planes
5 m plane at various depths
Brass plane

a-Velocity, v=8 m
b- " "=7 "
c- " "=6 "
d- " "=5 "
e- " "=4 "
f- " "=3 "
g- " "=2 "
h-L=10.00 m, T=500 mm
i-"= 7.50 ", "=375 "
j-"= 5.00 ", "=500 "
k-"= 5.00 ", "=250 "
l-"= 5.00 ", "=150 "
m-"= 5.00 ", "=100 "
n-"= 2.50 ", "=125 "
o-"= 5.00 ", "= 50 "
p-"= 1.00 ", "=200 "
q-"= 5.00 ", "= 25 "
r-"= 1.25 ", "= 62.5 mm
s-Froude, L=15.24 m
t- " "=8.534 "
u-Dresden, L=8.52 m
v- " "=4.60 "
w- " "=3.60 "
x-Froude, L=1.524 m
y-Dresden, L=1.60 m
z- " "=0.60 "
I-First experiment
II-Second experiment $u\zeta$

Fig. 9 Logarithmic diagram of results of all the experiments with planes (incl. Froude and Dresden experiments)

a=Resistance of a surface 1m wide and of various lengths.
b=Resistance of $1m^2$ on different portions of surface at various speeds$=\lambda v^{1.875} 0.875 L^{-0.125}$

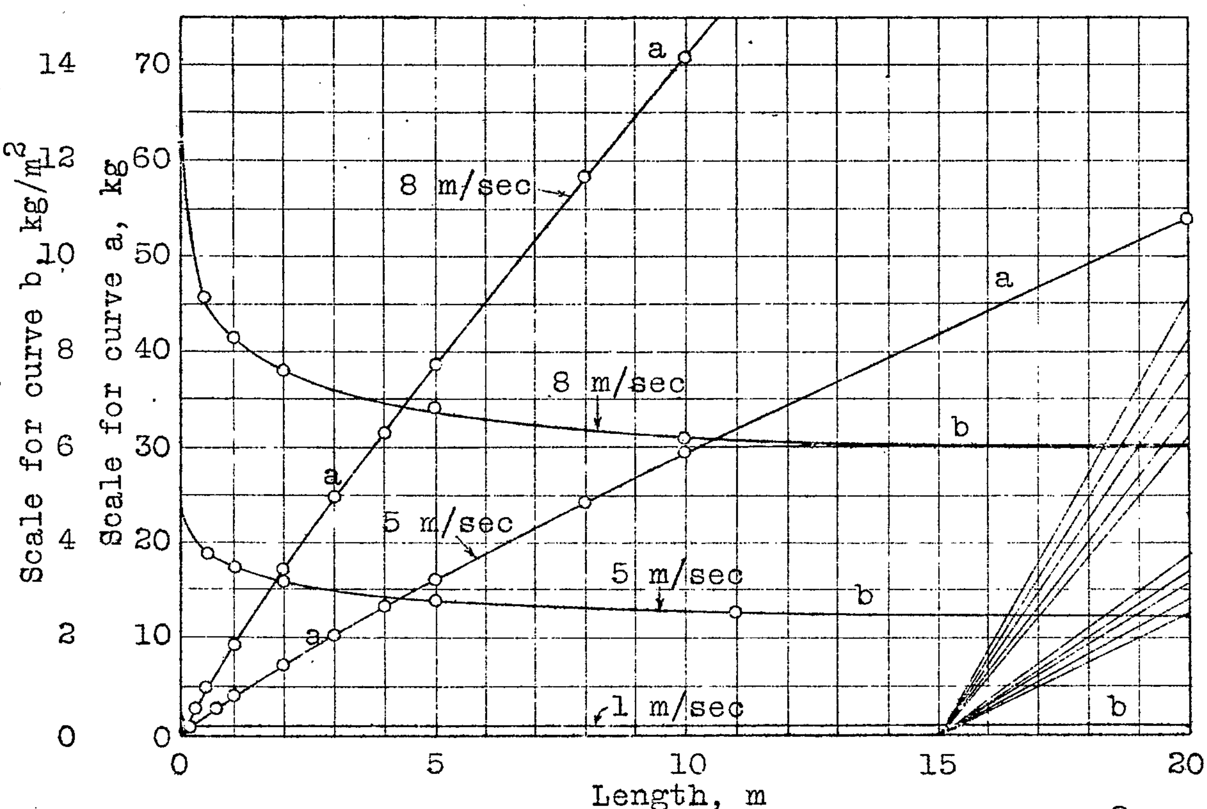

Fig. 10 Specific surface resistance (or resistance per m^2 of surface)

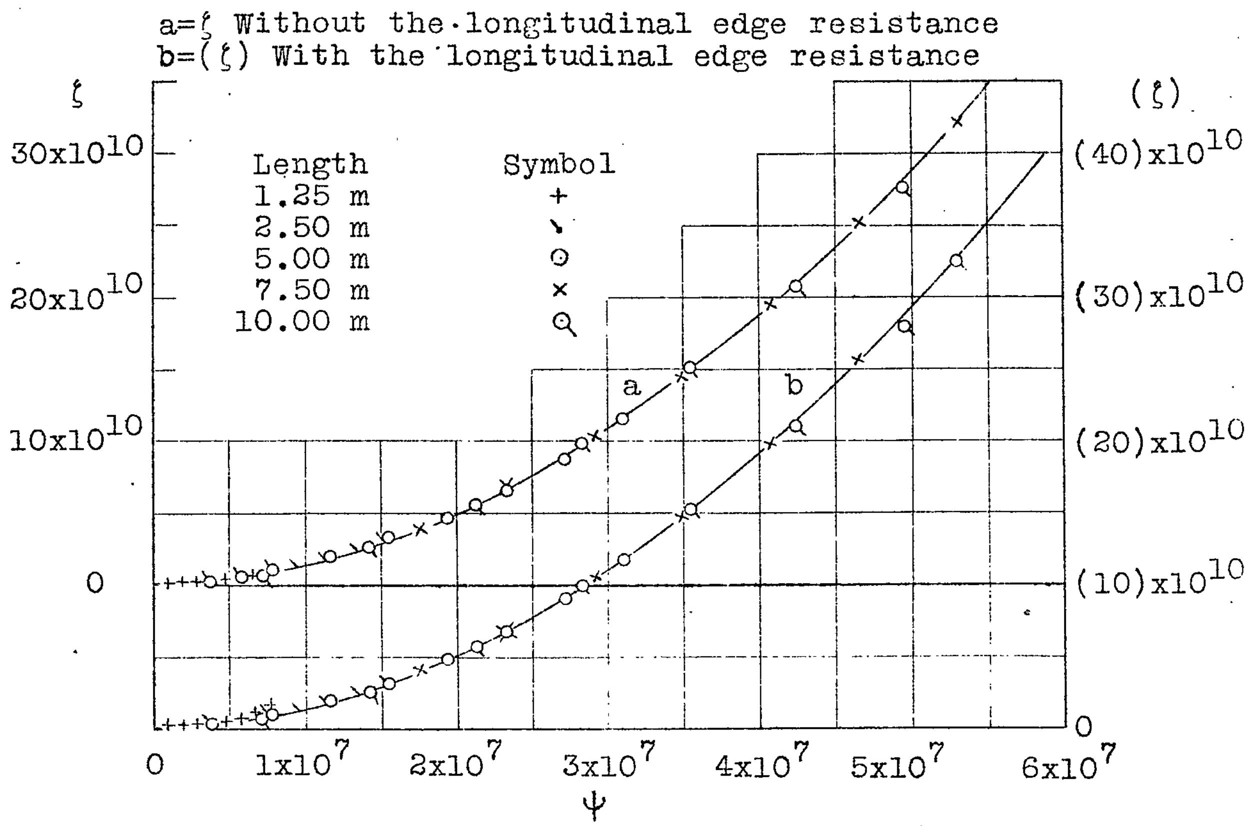

Fig. 11 Plotting of $\psi = \frac{vl}{\nu}$ as abscissas and $\zeta = \frac{k}{\rho V^2}$ as ordinates (k=surface resistance) for the planes tested.

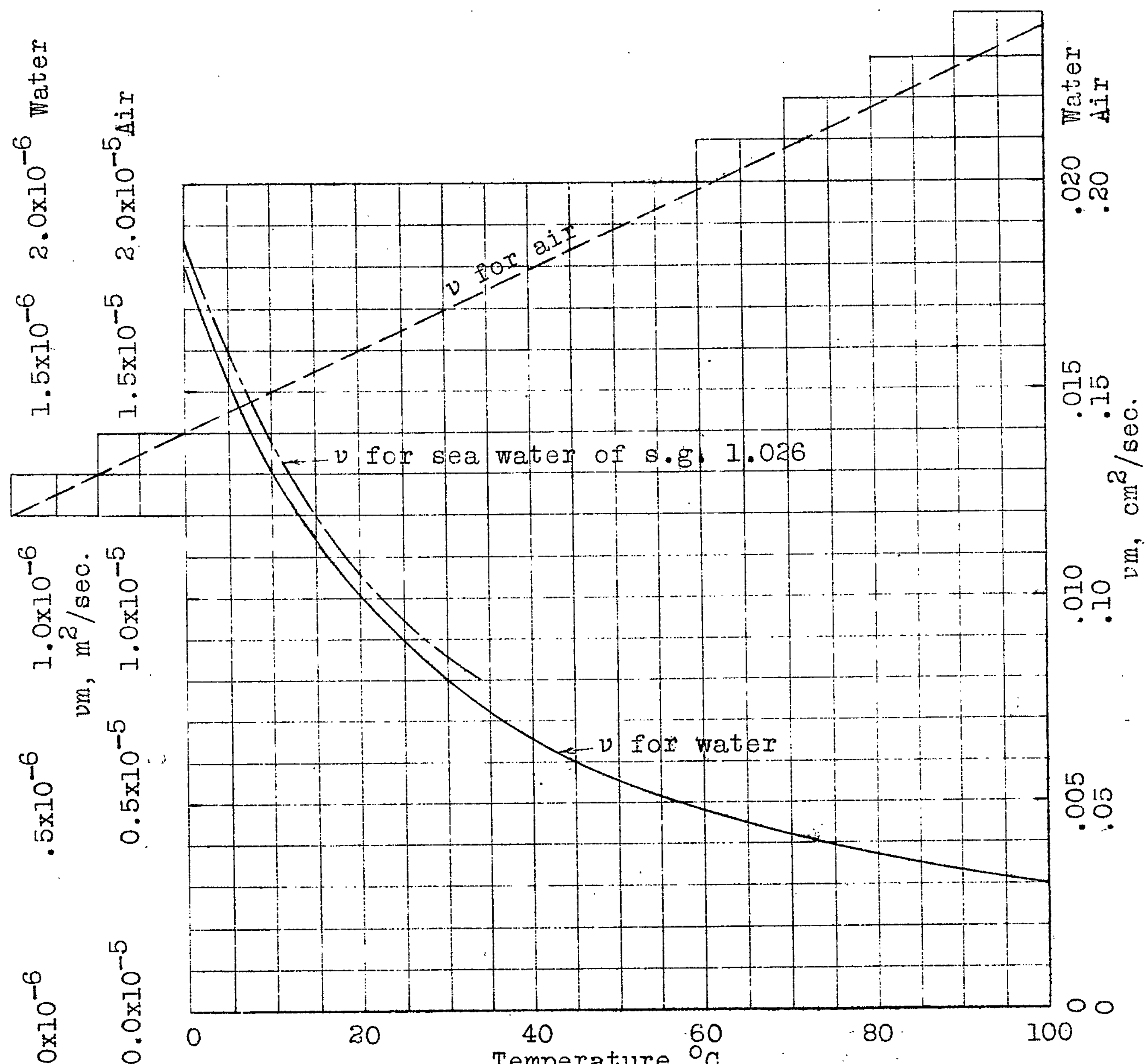

Fig. 12 ν plotted against temperature

a=Froude ($u^{1.825}$) $\lambda=0.213, l^{-0.130}$
b=Froude ($u^{1.825}$) $\lambda=0.161, l^{-0.027}$
c=Dresden ($u^{1.854}$) $\lambda=0.200, l^{-0.136}$
d=Vienna ($u^{1.875}$) $\lambda=0.193, l^{-0.125}$

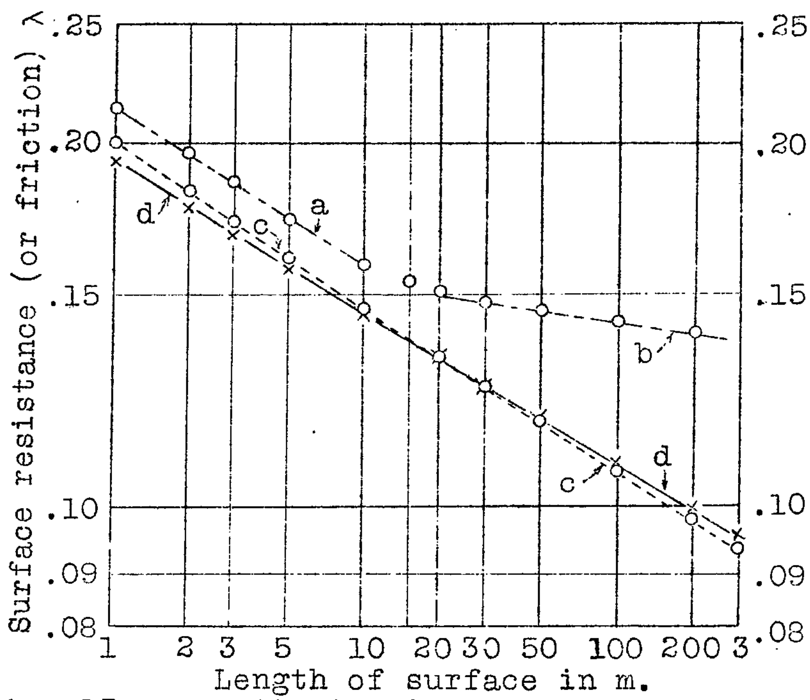

Fig. 13 Logarithmic diagram of surface resistance, λ